Addressing congestion on a rail network with a decentralised incentive scheme

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Introduction

During the last decade more and more countries have liberalised the traditional state monopoly on the railways. In Denmark this process started in 1999 for freight transport and in 2000 for passenger transport giving other than the national train operator access to the rail tracks. In early 2003 the British ARIVA started operating their first passenger trains on a route that was formerly operated by the (former) state railways – DSB. Some freight operators have worked on minor lines in the Southern part of Denmark for some time, but the main freight operator – Raillion – has operated on the main rail lines parallel to the passenger trains as a private company since 2001. Due to EU legislations national railways are also opened to international (transit) train operators. IKEA rail has used this opening to form their own rail company from transport between Älmhult in Sweden and Duisburg in Germany.

A problem that the regulator cannot control after the opening for competition is the quality the different train operators choose for their trains. The quality or the maintenance level chosen by the operators is important for the number of delays caused by trains that break down. This is a problem that will become more and more important as the number of different train operators that operate on the same rail lines increases. In Denmark this is to some extent already a problem due to the passenger trains and freight trains being operated by different companies. The solution that is used to avoid this problem is to keep different types of train separate from each other and to allow for larger time intervals (larger train channels) between different types of trains. However, as the pressure from more operators increases possibly due to increasing prices on road transport (road pricing), this instrument will become unavailable.

The problem we address here arises because high quality trains are more expensive to use than trains of lower quality. This gives the train operators incentives to keep a low standard of maintenance and a low quality of the trains. Hence, there is an inefficient level of delays caused by trains of (too) low quality.

The owner of the rail infrastructure is also a profit maximiser. To maximise profits he must sell as many train channels as possible. However, if there are to many delays in the infrastructure these obviously influence his ability to sell channels at a high price. Hence, he wishes to minimise the risk of delays. If the regulator has perfect information he can write a complete contract where quality is specified. But obtaining perfect information about the service operators' maintenance level and thus quality is very costly. This means that quality cannot be verified. It also implies that the infrastructure owner cannot impose an optimal regulatory tax on low quality.

In this paper we propose a decentralised incentive scheme that induces the operators to choose optimal quality. This is obtained without the very costly monitoring of the operators.

The next section introduces the model set-up. We focus on a closed rail-network and ignore the specific market structure of the network provider and the service operators. We take as given that delays are bad not only for the service operators, but also for the network provider. We also ignore the problems related to a network provider that is also a service provider (vertical integration).

An important element in the model is the information structure. The system can be operated in a world where the network provider has perfect information, but the system is more interesting in the case sketched out above with asymmetric information. The proposed system is aimed at solving exactly the problem of asymmetric (or imperfect) information for the regulator about the agents. In the case of perfect information it is as described above possible to write perfect contracts and thereby obtaining optimal use of the infrastructure.
A section discussing some practical issues of the proposed system follows the model description. This section also contains a discussion of similar regulatory schemes. We sum up our findings in the final section.

Model

There are $N<<\infty$ service providers denoted $i=1,\ldots,N$. The service providers' profit functions depend on the revenue (they get from e.g. ticket sales) r_i, direct transfers of money (e.g. through subsidies from the government), z_i, and the costs, $c_i(x_i,q_i)$. The costs are combined from a fee paid to the infrastructure owner to get access to the tracks, costs consisting of both operational costs and costs of quality provision. The quality costs arise due to maintenance, higher price for trains with high quality, but also the number of drivers and staff operating the trains influence costs and quality/risk of delays.
Service provider i has a profit function

$$\pi_i = r_i\left(x_i, x_j, q_{i,}, q_j\right) + z_i - c_i\left(x_i, q_i\right) \qquad (1)$$

A service provider can run passenger trains or freight trains. The revenues are negatively influenced by delays, which again are dependent on the number and quality, q_j, of trains, x_j, run by the other service providers. Revenues are positively dependent on the number and quality of trains run by the service provider himself.
An increase in x_i could also be more freight wagons or passenger coaches driven by the same locomotive. It could alternatively be a change in the quality of the trains (locomotive, coaches or wagons). The common feature of these different interpretations is that choices made by other service providers influence the revenue function. Here we choose to focus on the quality choices and assume that each service provider operate only one train. We will return to this below.
The profit of a service provider can also be positively related to the number and quality of the trains run by the competitors (the Mohring effect). An increase in the number or quality of the trains is an improvement of the service level for the customers this may increase the total number of customers. It does not necessarily have to be the service provider that increases the number or the quality of his trains that actually get (all of) these extra customers. The revenue of freight trains is possibly not positively influenced by an increase in the number or quality of trains by another service provider. This is from an analytical point of view the only difference between the two types of trains (or service provision). For the analysis it does not matter whether the impact from an increase in number or quality of trains by another service provider increase or decrease revenue for a specific service provider; i.e. whether the externality is positive or negative.

Sub-optimal levels of quality

To simplify we assume that the network provider is restricted (due to safety reasons) in the number of trains he can allow on the tracks. Hence, there are a fixed number of train channels. We assume that the network provider is interested in optimal use of his infrastructure. Optimality does not come by itself due to the uncontrollable quality choices made by the service operators using the network. That this is unwanted for him is because a high number of delays on his tracks influences the price he can set on train channels in subsequent renegotiations of the terms of access to his tracks. His interest is that the service providers use high quality trains so that the risk of delays is minimised. But quality is costly for the service providers:

$$\frac{dc_i(x_i, q_i)}{dq_i} > 0 \qquad (2)$$

where q is quality. The marginal cost of quality is increasing in quality: $\partial^2 c_i / \partial q_i^2 > 0$. This gives the service providers incentives to use low quality trains. Quality influences the revenue through a delay risk function γ where high quality gives a lower risk of delays than low quality does.

$$\frac{\partial r_i}{\partial q_j} = \frac{\partial r_i}{\partial \gamma_j(q_j)} \frac{\partial \gamma_j(q_j)}{\partial q_j} > 0 \qquad (3)$$

In (3) we have assumed that each service provider only operates one train. In our setting this means that the amount of transport x_j does not matter for the revenue. So to ease the notation we can set $x_j = \gamma_j(q_j)$. An equivalent and perhaps more realistic interpretation is that each train channel is distinguished, but a service provider can run trains in more train channels. Index j will in this interpretation correspond to a train channel and not the service provider operation the train.

Without being exposed to regulation each service provider maximises profit by the choice of quality. The choice of quality by the service providers is then given by

$$\frac{\partial r_i}{\partial x_i} = \frac{\partial c_i}{\partial x_i} \qquad (4)$$

This is not optimal. The network owner’s profit is dependent on the individual service providers’ profit. So to make things easy we assume that he wishes to maximise the sum of the service providers’ profits.

This is obtained when the following conditions hold

$$\frac{\partial r_i}{\partial x_i} + \sum_{j \neq i} \frac{\partial r_j}{\partial x_i} = \frac{\partial c_i}{\partial x_i}, \quad \forall i \qquad (5)$$

It is obvious from (4) and (5) that optimality without regulation can only occur when there is no externality (where choice of quality does not affect any other service provider or when the sum of positive and negative externalities cancel out).

This means that the problem is a standard externality problem. The traditional solution to this problem is that the regulator levies a Pigou tax equal to the marginal utility by all agents of the choice by a specific agent, $t_i^P = \sum_{j \neq i} \partial r_j \big/ \partial x_i$.

Information structure

Levying a Pigou tax means that the regulator must have perfect information about the exact cost and revenue functions of every service provider. This is not a likely situation. The service providers will try to conceal some of their information to get a strategic advantage in the contract negotiations with the network operator.

So we assume that the network operator is unable to collect perfect information about the quality of the service providers’ trains. The individual service providers are much better informed about the cost structure and quality of not only their own trains, but also the other service providers’ trains. This is not an unrealistic assumption. The service providers operate in the same market and can be expected to have good information about quality of different types of trains and also about the costs of maintaining high quality trains. Joining this with what they can observe from the operation of the trains we can suppose that the combination of this information is adequate.

Using the Compensation mechanism to regulate quality choices

The proposed solution to the network operator's problem is that he writes an additional incentive scheme into his contract with the service providers. He writes that they must participate in a Compensation mechanism game originally proposed by Varian (1994) as a general solution to an externality problem where participating agents have perfect information.

The Compensation mechanism consists of two stages; *an announcement stage* and a *choice stage*. Every service provider must in the announcement stage announce two things. They must announce how much they will subsidise the other service providers' investment in quality. We call this announcement by service provider *i,* s_j^i. They must secondly announce how much he should be subsidised for his own investment in quality, t_j^i.

The service providers maximise profit in stage two given the announced subsidies in stage one.

$$\max_{x_i} \pi_i = r_i(x_i, x_j) - \sum_{j \neq i} t_i^j x_j - c_i(x_i) + \sum_{j \neq i} s_i^j x_i - \sum_{j \neq i} \left(s_j^i - t_i^j\right)^2 \qquad (6)$$

The profit function now consists of three additional elements. The first addition is a deduction of the subsidies he must pay to the other service providers for the investment in quality they undertake, $\sum_{j \neq i} t_i^j x_j$. The second element is the subsidy he receives from the other service providers, $\sum_{j \neq i} s_i^j x_i$. The final addition is a penalty function increasing in differences in announcements.

If we combine the two stages we can simplify the decision problem to one where the service providers engage in the Compensation mechanism game, where they maximise profit using both quality and announcements of subsidies

$$\max_{x_i, t_j^i, s_j^i} \pi_i = r_i(x_i, x_j) - \sum_{j \neq i} t_i^j x_j - c_i(x_i) + \sum_{j \neq i} s_i^j x_i - \sum_{j \neq i} \left(s_j^i - t_i^j\right)^2 \qquad (7)$$

First order conditions for the service providers are

$$\frac{d\pi_i}{dx_i} = \frac{\partial r_i}{\partial x_i} - \frac{\partial c_i}{\partial x_i} - \sum_{k \neq i} s_i^k = 0, \quad \forall i \qquad (8)$$

$$\frac{d\pi_i}{dt_j^i} = \frac{\partial x_i}{\partial t_j^i}\left(\frac{\partial r_i}{\partial x_i} - \frac{\partial c_i}{\partial x_i} + \sum_{j \neq i} s_i^j\right) + \frac{\partial x_j}{\partial t_j^i}\left(\frac{\partial r_i}{\partial x_j} - t_i^j\right) = 0, \quad \forall i, j \qquad (9)$$

$$\frac{d\pi_i}{ds_j^i} = \frac{\partial x_i}{\partial s_j^i}\left(\frac{\partial r_i}{\partial x_i} - \frac{\partial c_i}{\partial x_i} + \sum_{j \neq i} s_i^j\right) + \frac{\partial x_i}{\partial s_j^i}\left(\frac{\partial r_i}{\partial x_j} - t_i^j\right) - 2\left(s_j^i - t_i^j\right) = 0, \quad \forall i, j \qquad (10)$$

where (8) is the second stage reaction function to the announced subsidies. (9) and (10) give the choice functions for the stage one announcements when the reaction in stage two is taken into account. Rearranging on the terms lead to $s_j^i = t_i^j$ and $t_j^i = \partial r_i / \partial x_j$, which implies that $s_i^j = \partial r_j / \partial x_i$ using this in (8) gives us

$$\frac{\partial r_i}{\partial x_i} + \sum_{j \neq i} \frac{\partial r_j}{\partial x_i} = \frac{\partial c_i}{\partial x_i} \qquad (11)$$

This is exactly equal to the condition (5) that ensures an optimal choice of quality by the service providers.

Some important issues

Why does the mechanism work?

What gives the result derived in the previous section are two things. First the punishment for announcing a subsidy, s_j^i less than what is demanded from the other service providers, t_i^j. This induces the service providers to announce what the others have announced, $s_j^i=t_i^j$. Secondly that offering a too large subsidy to the others leads to an oversupply of quality, which he must then pay a high subsidy to. On the other hand offering a too low subsidy leads to less quality than desired and with to many delays as a consequence. There is no additional profit to be earned by altering announcement when the marginal change in profit equals the announced subsidy. A more detailed analysis can be found in Kveiborg (2003).

Incomplete information

An important element in the mechanism is that the service providers have perfect and complete information about the other service providers. Even though the service providers may have good information – because the number of other service providers is well defined and countable, and because the service providers operate under similar conditions – there may be special characteristics within each service provider's operation that is only partly known. This obviously implies that the service providers cannot make precise matching announcements. To analyse how this influences the compensation mechanism it is necessary to assume an out-of-equilibrium behaviour of the service providers. One reasonable strategy is to use a Cournot best-reply strategy, where the service providers use the behaviour of the other service providers in the previous section. This is a situation that is analysed by Milgrom and Roberts (1991) who show that a best-reply strategy is consistent with adaptive learning. Often this is a reasonable behaviour.

To illustrate how a Cournot best reply strategy leads to optimality we describe the strategy through the two following discrete adaptations. First each service provider update his announcement of how much to subsidise the other service providers, s_j^i by announcing what they announced in the previous period:

$$s_j^i(\tau) = t_i^j(\tau - 1) \tag{12}$$

where τ is time period. Secondly each service provider realise that he has not maximised profit by his announcement of t_j^i. He then updates his announcement via $t_j^i = \partial r_i / \partial x_j$ derived from the first order conditions (8)-(10):

$$t_i^j(\tau) = t_i^j(\tau - 1) + \delta_j \left(\frac{\partial r_i}{\partial x_j} - t_j^i(\tau - 1) \right) \tag{13}$$

Stability is found by analysing the eigenvalues of the matrix, ***A***, of first order derivatives of this difference system for any pair *i-j*. The eigenvalues must be less than 1 to assure stability of the system of pairwise difference equations (12) and (13) (Sydsæter, 1990).

$$\mathbf{A} = \begin{bmatrix} \partial s_j^i(\tau) \big/ \partial s_j^i(\tau-1) & \partial s_j^i(\tau) \big/ \partial t_i^j(\tau-1) \\ \partial t_i^j(\tau) \big/ \partial s_j^i(\tau-1) & \partial t_i^j(\tau) \big/ \partial t_i^j(\tau-1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\delta_j & \delta_j \, \partial \big/ \partial t_i^j(\tau-1) \left(\frac{\partial r_i}{\partial x_j} \right) \end{bmatrix} \tag{14}$$

Kveiborg (2003) shows that for δ_j less than 1 and greater than 0 will the system of difference equations be stable. However, it can only be derived analytically when the influence from x_j on revenue is linear. Kveiborg (2003) further conjectures that this may actually also hold in more general non-linear contexts.

That the system is stable means that any deviation from optimality will over time evolve towards the optimality equilibrium. Hence, we conclude that the service providers even when they do not have perfect information about each other, will gradually learn what the optimal announcements are.

Contracts

It has already been mentioned above that it is not possible to write a perfect contract that takes all contingencies into account. Many of the uncertainties that could be written into the contract are non-verifiable. One such element is the choice of quality. Quality is in this context also understood as the general maintenance level of the trains. But it could also be the staff operating the trains. Less staff increases the risk of something delaying the trains. The service providers better know the relation between quality and maintenance costs etc.

The contracts between the network operator and the service provider specify the train channel, the type of train (passenger, freight, number of wagons, dangerous goods etc.) and other objective elements. However, the specific type of locomotive or wagon should not be specified to restrictive. It should be possible for the operator to change locomotive in order to optimise his operation (e.g. through a time schedule for the rolling stock etc.) or to have a specific locomotive taken out for repairs. Often the wagon is for specific use (bulk or container transport). It should therefore be possible to use the wagons suited for the specific type of goods that is conveyed on a specific train or departure. This further adds to the uncertainty of the network supplier.

The contracts contain an additional clause specifying the terms of a Compensation mechanism game that the service operators must participate in, and the time intervals of the game/announcements are also specified.

When a network operator writes contracts with service providers this is often a contract with a long time span. The contract with ARRIVA concerning the passenger train operations in the western part of Jutland is running for seven years. It is possible that contracts can be written for only a single train channel at a specific time. However, this requires vacant capacity, which is becoming more rare especially during the busy daytime and as demand for permanent channels increases.

Implementation of the Compensation mechanism game

Not all contracts are written for the same time periods. Hence, new operators will enter and leave the rail network. Moreover, the service operators will definitely change their operation (material and ways of supplying their transport service etc.) through the contract period. It is thus not enough to play the Compensation mechanism game only once. The Compensation mechanism game should be repeated at regular intervals; e.g. once a month.

Some time in advance of a new calculation of subsidies a list is sent out specifying the different service operators. Information about the known (contractual) specifics is supplied to all operators. They must then before a specific data finish their announcements of subsidies.

When announcements are made these are immediately available for all. The service operators can then make their choices of maintenance and quality of the trains they operate.

The process specified here gives the service operators time to adapt to the specified announcements. It is also imaginable that the process involves several rounds of announcements before the announced subsidies are actually implemented. This enables the service operators to change their announcements in such a way that they avoid paying penalties. This process is expected to converge as discussed above.

Is the Compensation mechanism an optimal instrument?

The Compensation mechanism is like any other (known) instrument not optimal. However, there are many advantages by using the Compensation mechanism in relation to the specified problem in relation to delays in rail transport.

In comparison with a road network we can consider a rail network as a much simpler problem. Not only because the network is much smaller, but also because the number of operators is smaller. In relation to this is the information possessed by the various agents (the network owner and the service providers). We expect the service providers to be better informed about the market and the other service providers than the network owner, as already discussed. The rail network using train channels can to a large extent be considered a closed system in which all agents are known. Moreover, many of the service providers' characteristics are well defined. Who operates in the different train channels, what general type of train do they operate, etc. Hence, the task of making announcements is manageable. However, there is one issue that should be mentioned. Diffusion of delays to the entire rail network is often the case. This can be verified by examining the weekly update of delays in the entire rail network. The graphical illustration published e.g. by the "Banestyrelsen" on their website clearly shows that when delays happen in one part of the network there is widespread pattern of delays (see www.banestyrelsen.dk/korevej/regular/pass). This implies that the service operators have to take into account (almost) all train channels in the network.

It should also be remembered that the train operators are not the cause of all delays. Signalling problems etc. also cause delays. Such delays are not included in the set-up described above. It is possible to design the Compensation mechanism such that the network operator as a cause of delays is also included. The problem with insufficient maintenance of the infrastructure is similar to the choice of quality by the service provider. Hence, by letting the network owner participate in a Compensation mechanism would solve this problem. Normally the problems of delays related to infrastructure are covered by the contract between network owner and service operator. However, due to asymmetric information this is also an imperfect contract, but in this context it is the lack of maintenance by the network owner that is the problem.

Other regulatory schemes - fees

There are not many possibilities in regulating the type of externality sketched here in other ways that similarly ensure optimal quality choices. A Pigouvian tax is an example, but it requires perfect and complete information to set such taxes, as already discussed.

Many of the contracts between network owner and service provider (passenger trains operated at specific time schedules) contain paragraphs specifying how many delays is accepted. If they do not meet the specified targets fines are issued. Similar fees can be specified for choices of quality. These fees for not meeting required levels of quality could be interpreted as a Pigouvian type of fee. The delay fees can also be used in relation to freight trains. Even though freight trains are not supposed to stop at train stations to pick up passengers and meet time schedules like passenger trains, they must still obey time schedules. The train channels constitute such time schedules. Hence, if a freight train does not run within its train channel this can be classified as a delay.

We have also discussed the problem of writing perfect contracts because quality is not verifiable at court. This makes it impossible to dictate quality choices.

The alternatives thus do not seem likely as alternatives. However, there are more issues involved here. Transaction costs are important to include. In the analysis above we have not addressed this issue directly. We argued that the service providers might have very good information about each other. However, such information does not come without some costs. The service providers must invest time and other resources in gathering this information. On

the other hand some of this information can be used for optimisation of their own operation. Hence, not all transaction costs are additional cost invoked on the service providers.
The problem is only relevant if transaction costs are larger than the inefficiency costs induced by not using the "optimal" instrument. It is difficult to access whether this is actually the case. We do not think that the issue is of considerable size, because monitoring the market is part of the work undertaken by the service providers. The argument is enforced also by the way concessions or accesses to the rail tracks are won. Namely by submitting offers to the network operator. It is necessary to monitor the market and the competitors in order to give the highest offer.

Similar mechanisms

The Compensation mechanism might have other uses in relation to the railways. We can possibly also use the Compensation mechanism to allocate access to the infrastructure. The set up is an infrastructure owner that has a certain amount of space available for allocating to the service providers. However, defined in this way infrastructure is a good sold in a market. Hence, an auction where access is given to the service providers with the highest bids can solve this problem. There is thus no need for a more complex mechanism in order to solve this allocation problem. This is the system adapted in the Danish liberalisation of rail provision.
Eyckmans (1997) applied the Compensation mechanism as a solution to international negotiations about greenhouse gas reductions. The set up bears many similarities with the rail track reductions considered in the present paper. He used the mechanism to implement a proportional cost share equilibrium (see Corchón, 1996 chapter 5 for more about cost-share equilibria). The special thing about Eyckmans approach is that he interprets an externality as a public good independent of its origin. This makes the mechanism somewhat simpler because there is no difference in the impact from the externality dependent on who is the producer of the externality. Hence, this reduces the number of announcements that must be made. Eyckmans furthermore interprets the announcements as subsidies given to the cost of reducing emissions in the other countries participating in the negotiations.
There are not many mechanisms in the literature aimed at the externality problem. Most mechanisms approach the public good problem and not the externality problem. Among the few suggested solutions is Mas-Colell and Silvestre (1989). Instead of implementing an optimal allocation of externalities in a conventional Nash or subgame perfect Nash equilibrium, which is the equilibrium concept used in the present paper, they introduce a new equilibrium concept: *Balanced Linear Cost Share Equilibrium*. The suggested equilibrium concept is somewhat artificial though, whereas the Nash equilibrium concept (no-one can improve his situation by unilaterally changing behaviour) is more intuitive. To use the Mas-Colell and Silvestre solution it is not only necessary to use the new equilibrium concept, it is also necessary to interpret the externality causing good in a special way.
Sandholm (2003) provides another approach aimed at solving externalities. He considers congestion type of externalities and uses an evolutionary approach where a dynamic adaptation procedure is explicitly used. The current behaviour of the externality generating individuals is used to set a price (tax) on the congestion making activity (car driving) he is able to demonstrate that the dynamic process converges to an optimal allocation. However, a problem in this approach is that the regulator has to monitor every individual continuously. This is a problem that is not found in the Compensation mechanism.
The final mechanism that we will mention here is an approach suggested by Dantziger and Schnytzer (1991). They use a stage mechanism to implement Lindahl prices and individual contributions to a public good. The stage mechanism bears many similarities with the

Compensation mechanism. However, the outcome is again optimal provision of a public good and not externalities.

Conclusion

From the discussion in the previous section it seems that the Compensation mechanism is the only probable alternative at present. However, we have introduced the Compensation mechanism as a solution for one specific example involving delay type of externalities (not necessarily congestion) in a relatively small network. Kveiborg (2003) analyses the same mechanism in relation to road transport and indicates that this specific problem is probably too large to be solved by a completely decentralised incentive based scheme. He also presents some methods for simplifying the mechanism resulting in a much more manageable size. He uses the fact that congestion can be considered as being independent of the contributor. The same approach can also be applied here. For an individual service provider it does not matter specifically who is the cause for the delay. However, it does matter whether an operator using the adjacent train channel or a channel further away measured in time or space causes the delay. It is also necessary to consider the individual service provider because the specific characteristic for this operator is what enables him to maximise profits by the choice of subsidy.

Using mechanisms is a new approach and it introduces a new way of considering regulation. A problem in this respect is that the regulator must give up tax revenue and control over the system. Due to this there is an inherent resistance to such a system. On the other hand we do see more and more examples where similar incentive schemes or mechanisms are used to implement optimal choices. One such example is auctions. This is a specific type of mechanism ensuring that a good is given to the one with the greatest benefit from the good.

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