Taxation, Time Allocation and Externalities

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Abstract

Using the approach introduced in Becker (1965) this paper derives rules for optimal taxation in the presence of externalities. The same was done in Kleven (2004) in the case where externalities were excluded resulting in the inverse factor share rule for optimal taxation. This rule states that fast cars should carry a lower tax rate than slow cars because of time savings. Including externalities modifies the result and gives a simple extension to the tax formulae. The results emphasize that taxation of externalities and revenue-generating taxation of goods should not be looked on separately.

1. Introduction

In many cities problems related to traffic congestion are increasing. As a result the politicians wish to regulate the traffic. It is therefore important to choose the right instruments so that the goals set up by the politicians are realized. Should one system be implemented or can other instruments achieve the same at lower costs? What problem is the instrument designed to address? Can a given instrument be used to generate public revenue? Are the chosen instruments politically feasible and how do they interact with the rest of the economy?

Often the concept of marginal cost pricing is mentioned as a way to internalize the external costs of transport and thereby induce an optimal usage of the transport infrastructure. But if a tax instrument is to be used it is important to know how the optimal tax scheme is to be designed. It is also important to know how people react to a given tax instrument and how this influences other parts of the economy.

In the theory of optimal taxation characterizations of the optimal tax scheme have been derived in different situations. Some of the best-known results are the Ramsey rule (Ramsey (1927)), the inverse elasticity rule (see for example Sandmo (1976) and Auerbach and Hines (2002)) and the Colett-Hague result (Corlett and Hague (1954)). One general conclusion from these rules are that the tax system should be constructed such that the distortions to the economy is minimized. Note that no distortion does not mean no effect on demand. Introducing taxes even in a lump sum way would introduce changes in the economy through the income effects. The point is that the substitution effects should be minimized. The reason for this is that when some goods are taxed and other goods are not the consumers change their consumption behavior away from the first best and is at the core of the discussion of the taxation of labor income where the tax raises the relative price of work and lowers the price of leisure inducing lower work participation and higher demand for leisure. This problem is essentially what is dealt with in Kleven (2004). He demonstrates that the inverse elasticity rule also emerges in the Becker setup though in a modified form named the inverse factor share rule. This rule states that commodities, which reduce time consumption, should be taxed less than other goods. The reason for this is that by making more time available by taxing time consuming activities the consumers will respond by working more and therefore reduce the distortion caused by the income tax. In the case of cars this would indicate that fast cars (sport cars) should be taxed less because they increase the time savings involved in transport. Kleven points out though that the conclusion might not be so robust if externalities are included which is the case considered in the present paper.

Another result from tax theory deals with taxation as a way to internalize externalities referring to these as Marginal Cost Pricing or Pigouvian Taxation (Pigou (1920)). The idea behind these taxes is that the externality comes from a misspecification of the price of the good in question and by imposing the right tax on the good the price failure can be corrected thereby internalizing the externality. A Pigouvian tax may seem very simple when looked upon in a world where the only goal is to internalize externalities. The presence of other taxes and the fact that the governments in general have to raise revenue to function complicates the problem. One must remember though that it is all the taxes in the economy that make up the tax system. It is therefore interesting to characterize the optimal tax system when externalities are present and the government has to raise revenue. This approach was taken by Sandmo (Sandmo (1975)) in the standard model for optimal taxation. Based on his analyses one could state that only the actions and goods that cause externalities should be subject to extra taxation. This is known as the additivity property or the "principle of targeting". A similar result emerges here when externalities are included.

This paper will use the approach introduced by Becker (1965) representing time explicitly in the utility function to derive formulas that describes the optimal tax rules in the presence of externalities thereby extending the results found in Kleven (2004). The extension makes the results found by Kleven less clear and emphasizes how externalities can be incorporated into the setup. Section 2 will present the model and derive a characterization for the optimal tax system. The generalization of this characterization makes it difficult to get clear-cut rules about the design of the tax system and section 3 therefore derives results, which gives more intuition. Section 4 will discuss the policy implications and the last section concludes

2. The model

We assume that there are N+1 commodities and H households in the economy. Each household has a utility function with all the usual conditions for continuity and differentiability given by

$$U_h = U_h(Z_h^0, Z_h^1, ..., Z_h^N) - \xi(\overline{Z}^N), \qquad h = 1, ..., H$$

where $\overline{Z}^N = \sum_{h=1}^{H} Z_h^N$ is the total consumption of good N in the economy and

$$Z_h^i = f^i(X_h^i, L_h^i), \qquad i = 0,..., N$$

represents the way good Z_h^i is produced in household h using one market good X_h^i, time L_h^i and production technology f^i. We assume the production technology to be Leontief and that every household uses the same technology in the production of the different goods. Intuitively this means that if a household wants to see a movie at a cinema they have to allocate the time required to see the movie and they have to purchase movie tickets. One could argue that more than one market good could be required in the production which would result in X being a vector of these market goods, but to keep things as simple as possible we here assume that only one market good goes into the production of every consumption good. The function ξ is assumed to be increasing and hold all the normal properties of continuity and differentiability. As a result the total consumption of good N decreases the utility of the households.
Assuming that the number of households H is large we make the standard assumption that the individual household behaves as if $\frac{\partial \overline{Z}^N}{\partial Z_h^N} = 0$. This assumption can be interpreted as if every household knows that it affects the total consumption of good N but regards its contribution as insignificant. Using this we can now formulate the optimization problem for household h as

$$\max_{X_h^0, X_h^1,...,X_h^N, L_h^0, L_h^1,...,L_h^N} \quad U_h(f_h^0(,L_h^0), f_h^1(X_h^1, L_h^1),..., f_h^N(X_h^N, L_h^N))$$

$$s.t. \qquad \sum_{i=0}^{N} P^i X_h^i = wN_h$$

$$\sum_{i=0}^{N} L_h^i + N_h = T$$

where w is the wage rate and assumed identical for all households, N_h is the amount of time spend on work for household h, P^i is the consumer price of market good X^i and T is the total time available to the household.
This description of the households shows that these are not only modeled as consumers but also as producers. Therefore we start by taking a closer look on the production process taking place inside the household. The household seeks to produce the good Z_h^i in the efficient way. This problem can essentially be seen as an attempt to minimize the production costs of every unit of Z_h^i. Letting the factor input coefficients a_{Li} and a_{Xi} be the input of L^i and X^i in the production process and assuming that households see P^i as fixed the households solves the following problem for every commodity Z^i

$$\min_{a_{Xi}, a_{Li}} \quad P^i a_{Xi} + a_{Li}$$

$$s.t. \quad f^i(a_{Xi}, a_{Li}) = 1$$

hereby finding the cheapest way to produce one unit of the consumption food Z^i. The solution is characterized by the unit cost functions $a_{Xi}(P^i)$ and $a_{Li}(P^i)$ describing the cost of producing one unit of Z^i measured in factor input. Using the solution to this problem, normalizing both wage rate w and total time T to 1 and realizing that the two constraints in the utility maximization problem are interdependent (through the variable N_h), we can restate the household's maximization problem as

$$\max_{Z_h^0, Z_h^1, \ldots, Z_h^N} \quad U_h(Z_h^0, Z_h^1, \ldots, Z_h^N)$$
$$s.t. \qquad \sum_{i=0}^{N} Q^i(P^i) Z_h^i = 1$$

where $Q^i(P^i) = P^i a_{Xi} + a_{Li}$ is the total cost of consuming one unit of Z^i. To see this remember that $a_{Xi} = \frac{X_i}{Z_i}$ and $a_{Li} = \frac{L_i}{Z_i}$ are constants due to the Leontief production technology. Adding the two constraints in the original problem utilizing the normalization of w and T we get the single constraint above. Note that $P^i a_{Xi}$ is the direct cost of using X^i as input and a_{Li} is the value of the time used for the production which equals the earnings lost due to lower working time. Therefore $Q^i(P^i)$ is the total cost of consuming one unit of Z^i and the constraint says that the total cost of consumption must equal full income, which is the market value of the time endowment.

Realizing that this essentially is a standard utility maximization problem the solution is well known and can be characterized by the factor demand functions $Z_h^i(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h)$ and the indirect utility function $V_h(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h)$ where y_h represents artificial non-labor income for household h and is given exogenously. Furthermore we know that Roy's Identity stating that

$$\frac{\partial V_h}{\partial Q^k} = -\lambda_h Z_h^k, \qquad k = 0, \ldots, N$$

and the Slutsky Equation stating that

$$\frac{\partial Z_h^k}{\partial Q^j} = \frac{\partial \tilde{Z}_h^k}{\partial Q^j} - \frac{\partial Z_h^k}{\partial y_h}$$

are valid where $\tilde{z}_h^k$ is the compensated demand for good z_h^k.

Having characterized the households behavior we now focus on the government. We assume that the government seeks to maximize a Bergson-Samuelson type social welfare function

$$W = W(\overline{V}_1, \overline{V}_2, \ldots, \overline{V}_H)$$

Because the government takes account of the externalities in the economy the indirect utility function the government considers has the following form

$$\overline{V_h}(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h) =$$
$$V_h(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h) - \xi\left(\sum_{h=1}^{H} Z_h^N(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h)\right)$$

Furthermore the government must raise a revenue G resulting in the governmental budget constraint

$$\sum_{i=0}^{N}\left(t^i \sum_{h=1}^{H} X_h^i\right) = G$$

where t^i is the tax rate set by the government. Often it is assumed that good 0 can not be taxed. One interpretation of this is that the government can tax goods consumed through the taxation of the input of X_h^i. But assuming that good 0 is pure leisure and thus having $a_{X0} = 0$ the government can not tax this good. In the case where the government can tax all goods it would be possible to introduce taxes in a first best way. We therefore assume that good 0 is untaxable.

Assuming that the production sector operates under constant returns to scale and that the markets are fully competitive the producer prices p^i for good X^i are fixed. Defining the tax rates as $t^i = P^i - p^i$ the government therefore has full control over the consumer prices through the tax rates and we can write the government's problem as

$$\max_{P^1, P^2, \ldots, P^N} \quad W(\{\overline{V}_h(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h)\}_{h=1}^H)$$

$$s.t. \quad \sum_{i=1}^{N}((P^i - p^i)(\sum_{h=1}^{H} a_{Xi} Z_h^i(Q^0(P^0), Q^1(P^1), \ldots, Q^N(P^N), y_h))) = G$$

Following Diamond and Mirrlees (1971) we define the social marginal utility of income β^h as

$$\beta^h = \frac{\partial W}{\partial \overline{V}_h} \lambda_h$$

where λ_h is the marginal utility of total income for consumer h. Utilizing this and using the Slutsky equation, Roy's identity and the symmetry of the Slutsky matrix we can now write the first order condition for the governments problem as

$$d_k = \frac{1}{\mu} \frac{\sum_{h=1}^{H} \beta^h Z_h^k}{\sum_{h=1}^{H} Z_h^k} - 1 + \frac{\sum_{i=1}^{N} t_i a_{Xi} \sum_{h=1}^{H} Z_h^k \frac{\partial Z_h^i}{\partial y_h}}{\sum_{h=1}^{H} Z_h^k}$$

$$- \frac{\sum_{h=1}^{H} \frac{\beta^h}{\lambda^h} (\sum_{\bar{h}=1}^{H} \frac{\partial \xi}{\partial Z_{\bar{h}}^N} \frac{\partial \tilde{Z}_{\bar{h}}^N}{\partial Q^k} - \sum_{\bar{h}=1}^{H} \frac{\partial \xi}{\partial Z_{\bar{h}}^N} Z_{\bar{h}}^k \frac{\partial Z_{\bar{h}}^N}{\partial y_{\bar{h}}})}{\mu \sum_{h=1}^{H} Z_h^k}, \qquad k = 1, \ldots, N$$

where

$$d_k = \frac{\sum_{i=1}^{N} \sum_{h=1}^{H} t_i a_{Xi} \frac{\partial \tilde{Z}_h^k}{\partial Q^i}}{\sum_{h=1}^{H} Z_h^k}$$

is the index of discouragement defined in Mirrlees (1976).

This formula characterizes the optimal tax system in the economy. Knowing that the compensated demand decreases when the price increases the discouragement index is negative (if the tax is positive). The right hand side tells us that if a good is demanded by households who are socially important (they have a high value of β^h) the discouragement should be reduced. Furthermore if changes in the demand for goods are highly sensitive to

changes in income (high $\frac{\partial Z_h^i}{\partial y_h}$) the tax on these goods should also be such that the discouragement is smaller.
To get some intuitive results the following section derives versions of some known tax rules known from the standard tax model.

3. Tax rules

In this section we will derive several known tax rules. Even though these rules are less general than the result found in the previous section they give more clear guidelines as to how the politicians should construct the tax system.

The Ramsey Rule

If we assume that there are no externalities in the economy the general result reduces to

$$d_k = \frac{1}{\mu}\frac{\sum_{h=1}^{H}\beta^h Z_h^k}{\sum_{h=1}^{H} Z_h^k} - 1 + \frac{\sum_{i=1}^{N} t_i a_{Xi} \sum_{h=1}^{H} Z_h^k \frac{\partial Z_h^i}{\partial y_h}}{\sum_{h=1}^{H} Z_h^k}, \qquad k = 1,\ldots,N$$

which (following Myles (1995)) we by defining $\overline{Z}^k = \frac{1}{H}\sum_{h=1}^{H} Z_h^k$ can be rewritten as

$$d_k = -(1 - \sum_{h=1}^{H} \frac{b^n}{H}\frac{Z_h^k}{\overline{Z}^k}), \qquad k = 1,\ldots,N$$

where $b^h = \frac{\beta^h}{\mu} + \sum_{i=1}^{N} t_i a_{Xi} \frac{\partial Z_h^i}{\partial y_h}$ is the social marginal utility of income defined in Diamond (1975). It is easy to see that if $b^h = b$ the expression simplifies to

$$d_k = b - 1, \qquad k = 1,\ldots,N$$

The optimal tax therefore reduces the compensated demand for all goods with the same proportion which is the Ramsey Rule.

The inverse factor share rule

To obtain the inverse factor share rule found in Kleven (2004) we take the government's maximization problem as a starting point. We assume that the government ignores distributional considerations and only seek to maximize the unweighted sum of household's utility. Furthermore we assume that all households are identical and that there are no externalities. The first order condition for the government's optimization problem can then be written as

$$\frac{\lambda - \mu}{\mu} = \sum_{i=1}^{N} \frac{\partial Z^i}{\partial Q^k} t_i a_{Xi} \frac{1}{Z^k}, \qquad k = 1,\ldots,N$$

If no cross-price effects are present in the economy and we let $\theta = \frac{\lambda - \mu}{\mu}$ we get the tax formulae

$$\frac{t_k}{P_k} = \frac{\theta}{\alpha_{Xk}\varepsilon_{kk}}, \qquad k = 1,\ldots,N$$

where $\alpha_{Xk} = \frac{P_k a_{Xk}}{Q^k}$ is the cost share of X^k in the price of Z^k and $\varepsilon_{kk} = \frac{\partial Z^k}{\partial Q^k}\frac{Q^k}{Z^k}$ is the own price elasticity of commodity k. This is the inverse factor share rule saying that goods which uses much time in household production should carry a lower tax rate than goods which primarily uses market produced commodities in the household production.

It is easy to see that the inverse elasticity formula is imbedded in this formulation. Letting $a_{Xk} = 1$ for the taxable goods the model reduces to the standard model used in the analysis of optimal taxation resulting in the inverse elasticity formula stating that goods with high own price elasticities should be taxed less in order to reduce the distortions caused by the taxation.

The additivity property

When externalities are present we can derive a result similar to the one found above. Again we take the governments maximization problem as a starting point, assumes that all households are identical and that the government maximizes the unweighted sum of households utility. This gives the first order condition

$$\frac{\lambda-\mu}{\mu} = -H\frac{\partial\xi}{\partial Z^N}\frac{\partial Z^N}{\partial Q^k}\frac{1}{Z^k}\frac{1}{\mu} + \sum_{i=1}^{N}\frac{\partial Z^i}{\partial Q^k} t_i a_{Xi}\frac{1}{Z^k}, \qquad k = 1,\dots,N$$

which gives the tax formulas

$$\frac{t_k}{P_k} = (1-\hat{\theta})\frac{-1}{\alpha_{Xk}\varepsilon_{kk}}, \qquad k = 1,\dots,N-1$$

$$\frac{t_N}{P_N} = (1-\hat{\theta})\frac{-1}{\alpha_{XN}\varepsilon_{NN}} + \hat{\theta}\frac{H\xi'}{\alpha_{XN}Q^N\lambda}$$

where $\hat{\theta} = \frac{\lambda}{\mu}$. For the goods not causing externalities the optimal tax is still determined by the inverse factor share rule. When externalities are present the inverse factor share rule still plays a rule but the tax rate now also takes account of the externality. It is seen that the extra term in the tax formulae enters additively. This additivity property was first noticed in Sandmo (1975) in a standard tax model with externalities and discussed further in Kopczuk (2003). The interpretation of the result is that when externalities cause consumers to ignore the true marginal costs of the goods they consume the tax problem can be separated into two parts. First a tax is used to correct the price so that the externalities are reflected in the price facing the consumers. Thereafter the normal tax rules are used to generate the required revenue. The final tax rates are seen to be a weighted sum of the two terms. Note that if the externality part of the tax is high enough the formulas describe how the surplus should be distributed turning the tax into a subsidy.

4. Policy implications in the transport sector and some guidelines

The tax rules found in the previous section help us to understand how the tax system should be designed. If distributional considerations are ignored the inverse factor share rule states that “Fast transportation should carry a lower rate of tax than slow transportation” (Kleven (2004)) which in the case of cars states that a sports car should be taxed less than a normal car. This conclusion is less clear when externalities are included because a faster car might cause externalities that a slow car does not (for example accidents and pollution). Furthermore we can see that if the choice of transport is highly sensitive to changes in its price the tax rate should be kept low in order to reduce distortions. If distributional considerations are included we see that the conclusion changes because sports cars are normally not used by households who are socially important.

We can summarize these guidelines by saying that

- goods which require large amounts of time for consumption should be taxed higher than other goods. This ensures that the distortions away from market produced goods to household produced goods are minimized. It is this distortion that is in the core of the discussion concerning the discouragement of labor supply caused by income taxation. In the case of car taxes slow cars should be taxed more heavily than fast cars.
- goods which have a high own-price elasticity should be taxed less in order to reduce the distortions in the demand for the goods. Typical necessities have low own-price elasticity and should therefore carry a high tax rate. This often clashes with the distributional wishes of the governments.
- if the goods are consumed primarily by socially important households (normally poor households) the tax rate should be kept low. This is the normal idea of redistribution between income groups and is often seen as one of the purposes of the tax system.
- if the consumption of a good causes externalities a tax should be levied on the good making sure that these are internalized without thinking about the governments budget requirements. When the externalities are internalized taxes (or subsidies) are used to bring the budget in balance. This is the additivity property.

We can see that some of these guidelines point in opposite directions especially if distributional considerations are included. If the tax revenue raised from the pure Pigouvian tax is exactly equal to the revenue required by the government (that is $\hat{\theta} = 1$) no further taxes are needed and the tax system is actually first best. But since this is highly unlikely to be the case the tax system will probably be of the second best nature. One of the goals of the government should therefore be the reduction of the distortions in the economy in general leading to the discussion the double dividend (see Goulder (1995) for a discussion).

5. Conclusion and possible extensions

In this paper we have presented a model for household behavior where time enters the utility function directly. Since the consumption of time is very important in the transport sector the approach is a natural extension to the traditional microeconomic when this sector is being modeled. The method seems very natural if one thinks about the processes taking place in the society and it is therefore important to explore the properties of the model.

We have extended the results by Kleven and included externalities in the approach. We showed that the tax formulas emerging resemble those found by Sandmo and we therefore conclude that the additivity property survives in this new setup. Furthermore we make it possible to see how distributional questions will affect the tax system.

Applied to car transport this means that a fuel tax introduced to internalize pollution might be reduced by the fact that the general tax introduced after the externalities has been internalized turn out to be lower than expected. This is due to the inverse factor share rule saying that if the uses of the fuel saves time it should in general carry a lower tax rate. Whether or not the tax should be "high" or "low" depend on the magnitude of these two effects. It is important to realize though that the time savings involved in the activities influence the magnitude of the distortions in the economy and therefore also the level of the optimal tax rate.

The model presented here should be generalized in several ways. The modeling of the externalities in a separable way could be criticized and alternative ways of modeling this will be subject to future research. Furthermore to assume that all households are identical or that they earn the same wage might seem unrealistic and the assumption that all households have the same technologies available to them could also be questioned. In spite of this we believe that the insights from the model are valuable.

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