Optimal taxation and congestion externalities in a model of household time allocation

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1. Introduction

In many major cities traffic related externalities are growing with pollution, accidents, noise and congestion as the major causes for concern. The externalities are often divided into atmospheric type and congestion type with traffic-congestion being the major contributor to last group. Removing the problems with negative externalities altogether seems unrealistic and the best the policy makers can hope for is to reduce the externalities to levels acceptable by the public.

If one looks at the transport sector it is easily seen that time is a very important factor in everything that goes on. This is because transport itself almost never is consumed alone. Transport is part of other activities or one could say that transport is only conducted in order for other activities to be made possible.

In the economic literature the link between activities and the use of time was modeled explicitly in Becker (1965). Here it is assumed that the consumption of market produced commodities alone is not the source of utility. Instead the households produce consumption goods by combining household time and market produced commodities and it is these goods that give utility. This way of thinking is in line with the ideas behind activity based transport modeling and some papers have utilized the Becker-like approach when valuing time in transport (e.g. DeSerpa (1971), Jara-Diaz (2003)).

The tax system is one of the instruments a government have when trying to control the externality problem and the optimal tax system for the transport sector has been analyzed extensively in the literature (see for example Mayeres & Proost (1997), Mayeres & Proost (2001), Parry & Bento (2002) and De Borger & Van Dender (2003)). Unfortunately all these approaches fail to take account of the insights pointed out in Becker (1965) that the allocation of household time matters. A recent paper (Kleven (2004)) has shown that the rules for optimal taxation needs modification if time is implemented (using the Becker approach). However, Kleven (2004) does not deal with externalities which are crucial if the theory is to be applied in the transport sector. This problem was addressed in Nielsen and Pilegaard (2004) in the case of atmospheric externalities. In the present paper we implement congestion externalities in the model, which explicitly model the use of time and to derive the rules for optimal taxation.

The paper proceeds as follows. Section 2 presents the model and describes the behavior of the households, the government and some useful mathematical derivations. Section 3 derives rules for optimal taxation and section 4 concludes.

2. The Model

We assume that there are N+1 commodities and H households in the economy. Households do not consume commodities bought in the market directly. Instead they undertake a production where market produced commodities **X** and household time **L** is used to produce consumption goods **Z** using a production process **f** (this follows Becker (1965)). The government seeks to maximize a Bergson-Samuelson type social welfare function $W=W(V^1,..., V^H)$ where the utility of each household enters the social welfare function and it has to raise a revenue R by taxing marked produced commodities **X**.

2.1 The households

To ensure the existence an optimum each household has a utility function with standard properties given by

$$U^h = U^h(Z_0^h, Z_1^h, ..., Z_N^h, G), \qquad h = 1, ..., H$$

where $Z_i^h = f^i(X_i^h, L_i^h)$, i=0,...,N represents the way good Z_i^h is being produced in household h using one market good X_i^h, time L_i^h and production technology f^i. We let $G = G(\sum_{h=1}^{H} Z_N^h)$ describe the externality in the economy. We restrict the analysis to negative externalities (thus $\partial U / \partial G < 0$ and G'>0). It is assumed that it is the total consumption of Z_N and not only the consumption of market commodity X_N that causes externalities. Considering transport as the main example in this paper it is the intuitive choice. It is assumed that the production technology is Leontief and that every household uses the same technology in the production of Z_i (for a discussion of the fixed coefficient technology see DeSerpa (1971) and DeSerpa (1975)). Intuitively this means that if a household want to go on a trip across the Great Belt in Denmark they have to buy a ticket and allocate time for the trip. One could argue that more than one market good could be required in the production which would result in X being a vector of these market goods, but to keep things as simple as possible we here assume that only one market good goes into the production of every consumption good. The households are restricted in their choices by time and money. Letting P_i and w^h represent market prices for commodity i and household wage they therefore seeks to maximize U^h subject to the constraints

$$\sum_{i=0}^{N} P_i X_i^h = w^h T^h$$

$$\sum_{i=0}^{N} L_i^h + T^h = \bar{T}$$

where T^h is the time allocated to work and $\overline{T}$ is the total time available to the household. Realizing that the constraints are interdependent through T^h we utilize the Leontief technology and write

$$\sum_{i=0}^{N} Q_i^h(P_i, w^h) Z_i^h = w^h \overline{T}$$

where

$$Q_i^h(P_i, w^h) = P^i a_{Xi} + w^h a_{Li}, \qquad i = 0, ..., N$$

with $a_{Xi} = X_i^h / Z_i^h$ and $a_{Li} = L_i^h / Z_i^h$. Note that these coefficients are fixed and identical for all households since they face the same production technologies. The simplified households optimization problem are thus given by

$$\max_{Z_h^0, Z_h^1, ..., Z_h^N} \quad U^h(Z_0^h, Z_1^h, ..., Z_N^h, G))$$

$$s.t. \qquad \sum_{i=0}^{N} Q_i^h(P_i, w^h) Z_i^h = I^h$$

$$G = G(\sum_{h=1}^{N} Z_N^h)$$

where $I^h = w^h \overline{T}$ is the households full income. Note that $Q_i^h(P_i, w^h)$ is the total opportunity cost of consuming one unit of Z^i for household h consisting of the price of marked produced commodities used in the production process and the value of foregone earnings. Assuming that the number of households H is large, we make the standard assumption that the individual household behaves as if $\partial G / \partial Z_N^h = 0$ which means that the households either think that their behavior does not affect the externality or that they may realize that they affects it but regards its contribution as insignificant. This allows us to characterize the optimal solution by the indirect utility function V^h and the ordinary demand functions for Z_i^h. We write these as

$$V^h(Q_0^h, Q_1^h, ..., Q_N^h I^h, G)$$

$$Z_i^h(Q_0^h, Q_1^h, ..., Q_N^h I^h, G), \qquad i = 0, ..., N$$

Letting $\tilde{Z}_k^h$ represent the compensated demand for good k we know that the Slutsky equation and Roy's Identity are given by

$$\frac{\partial Z_i^h}{\partial Q_k^h} = \frac{\partial \tilde{Z}_i^h}{\partial Q_k^h} - Z_k^h \frac{\partial Z_i^h}{\partial I^h}, \qquad i, k = 0, ..., N$$

$$\frac{\partial V^h}{\partial Q_k^h} = -\lambda^h Z_k^h, \qquad k = 0, ..., N$$

where λ^h is the Lagrangian multiplier associated with the budget constraint of the simplified household optimization problem thus representing the marginal utility of extra income for household h.

2.2 The Government

Assuming that the production side of the economy operates under constant returns to scale and is fully competitive the producer prices for commodities are fixed and given by p^i. Defining the tax on commodities as

$$t^i = P^i - p^i, \qquad i = 0,...,N$$

thus gives the government full control over the price vector for commodities. Knowing the households problem the government thus seeks to solve

$$\max_{t^0,t^1,...,t^N} \qquad W(\{V^h(Q_0^h, Q_1^h,...,Q_N^h I^h, G)\}_{h=1}^H)$$

$$s.t. \qquad \sum_{i=1}^{N} t^i (\sum_{h=1}^{H} a_{Xi} Z_i^h (Q_0^h, Q_1^h,...,Q_N^h I^h, G)) = R$$

where we assume that a_{X0}=0 so that good 0 becomes pure leisure and thus untaxable (see Munk (2002) for a discussion of pure leisure). With the assumption of pure leisure in the model we can no longer tax in a first-best way and the optimal tax system therefore becomes second-best. With the assumptions made the characterization of the optimal tax system will be given by the first order conditions to the governments' optimization problem.

2.3 Helpful derivations and definitions

Before the rules for optimal taxation is derived it is useful to make some derivations and definitions. First we need to derive an expression for $\partial G / \partial P_k$, which is the effect of a price change in the level of the externality. Since the externality is caused by the consumption of good N we divide the effect of a price change on the demand for this good in the direct price effect and an externality effect. This gives

$$\frac{\partial Z_N^h}{\partial P_k} = \frac{\partial Z_N^h}{\partial Q_k^h}\Big|_G \frac{\partial Q_k^h}{\partial P_k} + \frac{\partial Z_N^h}{\partial G}\frac{\partial G}{\partial P_k}$$

The change in the level of the externality as a result of the change in P_k is given by

$$\frac{\partial G}{\partial P_k} = G' \sum_{h=1}^{H} \frac{\partial Z_N^h}{\partial P_k}$$

Combining these expressions gives

$$\frac{\partial G}{\partial P_k} = \xi G' a_{Xk} \sum_{h=1}^{H} \frac{\partial Z_N^h}{\partial Q_k^h}\Bigg|_G$$

where

$$\xi = \frac{1}{1 - G' \sum_{h=1}^{H} \frac{\partial Z_N^h}{\partial Q_k^h}}$$

is the externality feed-back effect which is in line with the feedback-effect in Mayeres & Proost (1997). The role of the feedback-effect in the optimal tax problem is clear. Since we seek a way to internalize the externality and also raise a certain amount of revenue the introduction of an environmental tax will change the level of the externality since the households price vector is changing. This change in the price vector will cause a change in the households' behavior thus changing the externality level which again will change the behavior of the households. It is this effect that is called the feedback-effect. The intuition becomes very clear if traveling by car in a congested city is considered. Imagine that the government seeks to solve the congestion problem by introducing road pricing. Making it more expensive in monetary terms to drive in the city will make some drivers stay out of the city thus reducing the level of congestion. The reduction of congestion will make car transport more attractive to other car users thus raising the level of congestion. The feedback-effect captures this and is thus very important to include if one wish to find the optimal level of taxation in an economy with congestion externalities.

Letting μ be the Lagrangian multiplier associated with the government budget constraint we follow Diamond and Mirrlees (1971) and Diamond (1975) and define the social marginal utility of income β^h and the net social marginal utility of income b^h for household h as

$$\beta^h = \frac{\partial W}{\partial V^h}\lambda^h$$

$$b^h = \frac{\beta^h}{\mu} + \sum_{i=1}^{N} t_i a_{Xi} \frac{\partial Z_i^h}{\partial I^h}$$

We see that β^h is the direct social value of increasing the budget for household h. But an increase in the households' budget will result in a change in the household behavior and thus the tax payments. The second term in the definition of b^h captures this effect and it is therefore the net social marginal utility of income for household h. Note that β^h is measured in household income and b^h is measured in government revenue.

3. Tax Rules

To derive the optimal tax rules in this economy we assume that the first order conditions for the government's optimization problem characterizes the optimal tax structure (see Myles (1995) chapter 4 for a discussion of this assumption). The Lagrangian is given by

$$L = W(V^1, V^2, \ldots, V^H) + \mu\left(\sum_{i=1}^{N} t^i \sum_{h=1}^{H} a_{Xi} Z_i^h - R\right)$$

which can be shown to give the first order conditions for the optimal tax

$$\sum_{h=1}^{H} Z_k^h (b^h - 1) = \sum_{i=1}^{N} t_i a_{Xi} \sum_{h=1}^{H} \frac{\partial \tilde{Z}_i^h}{\partial Q_k^h} + \sum_{h=1}^{H} \left(\frac{\left(\frac{\partial W}{\partial V^h}\right)}{\mu} \frac{\partial V^h}{\partial G}\right) \xi G' \sum_{\bar{h}=1}^{H} \left.\frac{\partial \tilde{Z}_N^{\bar{h}}}{\partial Q_k^{\bar{h}}}\right|_G + \left(\sum t_i a_{Xi} \sum_{h=1}^{H} \frac{\partial Z_i^h}{\partial G}\right) \xi G' \sum_{\bar{h}=1}^{H} \left.\frac{\partial \tilde{Z}_N^{\bar{h}}}{\partial Q_k^{\bar{h}}}\right|_G$$

for all k=1,…,N. We now define γ as the marginal cost to society of a change in the externality level G. This consists of the direct welfare effect $\sum_{h=1}^{H} \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial G} \frac{1}{\mu}$ which measures the welfare loss to society in governments' revenue and an indirect revenue effect $\sum_{i=1}^{N} t^i a_{Xi} \sum_{h=1}^{H} \frac{\partial Z_i^h}{\partial G}$ which is the change in revenue due to changes in the tax payments caused by the externality. We therefore have that

$$\gamma = \sum_{h=1}^{H} \frac{\partial W}{\partial V^h} \frac{\partial V^h}{\partial G} \frac{1}{\mu} + \sum_{i=1}^{N} t^i a_{Xi} \sum_{h=1}^{H} \frac{\partial Z_i^h}{\partial G}$$

and we can express the first order condition as

$$\sum_{h=1}^{H} Z_k^h (b^h - 1) = \sum_{i=1}^{N} t_i a_{Xi} \sum_{h=1}^{H} \frac{\partial \tilde{Z}_i^h}{\partial Q_k^h} + \gamma \xi G' \sum_{\bar{h}=1}^{H} \frac{\partial \tilde{Z}_N^{\bar{h}}}{\partial Q_k^{\bar{h}}}\Bigg|_G$$

This version of the first order condition will be the starting point in the rest of the paper.

3.1 Inverse elasticity rule and the inverse factor share rule

The inverse elasticity rule (see Sandmo (1976) for an introduction) emerge in its most simple form when we assume that all households are identical, no compensated cross-price elasticitites exist, that no externalities are present (that is G=0) together with a_{Xk}=1, k=1,…,N. This reduces the optimal tax rules to

$$\frac{t_k}{P_k} = \frac{\dot{\theta}}{\tilde{\varepsilon}_k}, \qquad k = 1, ..., N$$

where $\dot{\theta} = \frac{\lambda - \mu}{\mu} + \sum_{i=1}^{N} t_i \frac{\partial Z_i}{\partial I}$ is a constant since it is independent of k and $\tilde{\varepsilon}_k$ is the compensated own price elasticity for good k. The interpretation of the inverse elasticity rule is straightforward. We know that introducing taxes in the economy distort the price vector. This distortion will change the consumption pattern so that the real cost to society of raising a revenue R is larger than R. The optimal tax system should therefore be constructed in such a way that this cost of raising government revenue is reduced and this is done by taxing commodities with low compensated elasticity at higher rates than commodities with high elasticities.

To derive the inverse factor share rule (Kleven (2004)) in its simplest form we allow a_{Xk} to differ from 1. In this case we can write

$$\frac{t_k}{P_k} = \frac{\dot{\theta}}{a_{Xk}\tilde{\varepsilon}_k}, \qquad k = 1, ..., N$$

As pointed out in Kleven (2004) this rule is very intuitive since the distortions caused by taxation often comes from the reduction in working time and increase in leisure time. In the present setup the distortion comes from the fact that households switch from market use of time to non-market use of time. That is, the distortion is caused by the households increasing their level of time used for leisure, or more correctly they increase their use of time spend inside the household thus avoiding the tax payments. If we accept that even activities inside the household often requires some level of marked produced commodities (which is one of the key assumptions here) the inverse factor share rule says that those commodities which are very time consuming in the household production process should be taxed at a higher rate than less time consuming commodities. Applying this to the transport sector it says that two modes which only differ in their speed should be taxed such that the fast mode is taxed less than the slow mode. Unfortunately the possibility of negative externalities is ignored and this is also recognized by Kleven. Since negative externalities are one of the main reasons for introducing road pricing we have to extend the model to include these if we want to apply optimal tax theory it in the transport sector.

3.2 Ramsey Rule

The Standard Ramsey rule for optimal taxation emerges in its simplest form if we assume that all households in the economy are identical (that is b^n=b) and ignore the externalities (G=0). In that case we can define Mirrlees Index of Discouragement d_k (Mirrlees (1976)) as

$$d_k = \frac{\sum_{i=1}^{N} t_i a_{Xi} \frac{\partial \tilde{Z}_k}{\partial Q_i}}{Z_k}$$

The index of discouragement for good k tells how much the compensated demand for this good is changed if the price vector is reduced as a result of a change in the tax system. Using the definition of d_k the first order condition can be written as

$$d_k = (b-1)$$

This rule calls for tax rates to be set at levels which ensure that the proportionate reduction in the compensated demand for all taxable goods is identical which is the Ramsey rule.

3.3 Additivity property

The additivity property derived in Sandmo (1975) also emerges in the present setup under the assumption of no cross-price elasticities. With this assumption the first order condition reduces to

$$\frac{t_k}{P_k} = \frac{\sum_{h=1}^{H} Z_k^h (b^h - 1)}{\sum_{h=1}^{H} \alpha_{Xk} Z_k^h \tilde{\varepsilon}_k^h}, \qquad k = 1, \ldots, N-1$$

$$\frac{t_k}{P_k} = \frac{\sum_{h=1}^{H} Z_k^h (b^h - 1)}{\sum_{h=1}^{H} \alpha_{Xk} Z_k^h \tilde{\varepsilon}_k^h} + \gamma \xi G' \frac{\sum_{\bar{h}=1}^{H} \left. \frac{\partial \tilde{Z}_N^{\bar{h}}}{\partial Q_k^{\bar{h}}} \right|_G}{\sum_{h=1}^{H} \alpha_{Xk} Z_k^h \tilde{\varepsilon}_k^h}, \qquad k = N$$

The interpretation of these tax rules are not straight forward, but essentially it separates the Ramsey part of the tax problem from the Pigouvian part, stating that firstly the taxes should be set so that the externalities are internalized and secondly the taxes should be adjusted to meet the revenue requirement in the most efficient way. In the special case of identical households and additive separable externalities (which eliminates the feedback-effect) these tax rules simplifies to the results from Nielsen and Pilegaard (2004) giving

$$\frac{t_k}{P_k} = \left(\frac{\lambda - \mu}{\mu}\right) \frac{1}{\alpha_{Xk} \tilde{\varepsilon}_k}, \qquad k = 1, \ldots, N-1$$

$$\frac{t_k}{P_k} = \left(\frac{\lambda - \mu}{\mu}\right) \frac{1}{\alpha_{Xk} \tilde{\varepsilon}_k} + \frac{HV'}{\mu Q^k} \frac{1}{\alpha_{Xk}}, \qquad k = N$$

which allows us to gain some new theoretical insight into the optimal tax problem in the transport sector. The formulas for optimal taxation consist of three elements. The first $(\frac{1}{\alpha_{Xk}})$

is the inverse factor share rule and the second ($\frac{1}{\tilde{\varepsilon}_k}$) is the inverse elasticity rule both discussed above. The last element ($\frac{HV'}{\mu Q^k}\frac{1}{\alpha_{Xk}}$) is the additive term on the externality generating good. With no explicit modeling of time (e.g. having $\alpha_{Xk}=1$ for all k) it is just the standard additive term found in Sandmo (1975) saying that the extra tax should be set according to the principle for Pigouvian taxation. This no longer holds as the term $\frac{1}{\alpha_{Xk}}$ enters the formula. The tax rule states that the social planner has to account for the time allocation by subsidizing time saving activities even if these activities generate negative externalities.

Implications

Using the rules from section 3 as a guide for the design of the tax system several insights emerge. We see that if no compensated cross-price effects exist the simple inverse elasticity rule and the inverse factor share rule apply together with the additivity property. Furthermore the fact that we are having congestion externalities in the economy should not affect the efficiency part of the tax rules. It only affect the externality (or Pigouvian) term of the tax formula. With cross-price effects present in the model we have to use the more complicated Ramsey tax rule. This says that the optimal tax rates should be set so that the reduction in compensated demand is equal for all goods.

4. Concluding remarks

It was shown that the rules for optimal taxation with externalities need modification when time allocation is modeled explicitly. This insight is especially relevant for the transport sector. The conclusion is that a transport mode which saves time should be taxed less than other modes even if these generate the same or possibly even a higher level of atmospheric externality. The obvious example is the discussion of taxes on busses and cars. When ignoring the time allocation the transport modes should carry the same level of externality tax if their external damage costs are the same. Here this conclusion is no longer valid per se. If the car is faster than the bus it should carry a lower externality tax. The argument that cars, due to their higher environmental damage, should be taxed higher than busses thus looses some of its appeal in the present setup and since cars actually might save time they should have their corrective tax reduced to a level below the pure Pigouvian tax rate.

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