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A Polynomial Estimate of Railway Line Delay

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Abstrakt

Railway service may be measured by the aggregate delay over a time horizon or due to an event. Timetables for railway service may dampen aggregate delay by addition of additional process time, either supplement time or buffer time. The evaluation of these variables has previously been performed by numerical analysis with simulation. This paper proposes an analytical estimate of aggregate delay with a polynomial form. The function returns the aggregate delay of a railway line resulting from an initial, primary, delay. Analysis of the function demonstrates that there should be a balance between the two remedial measures, supplement and buffer. Numerical analysis of a Copenhagen Sbane line shows that the polynomial function is valid even when theoretical assumptions are violated.

Introduction

Operational stability and robustness are quite important for railway transport. Not only are the passengers or users of the service sensitive to these measures of quality, but railways are usually integrated systems or networks, and failures at one location of the system affect other locations and services, sometimes quite catastrophically. A railway network planner is faced with many decisions about what quality of service to provide and what resources to allocate to deliver this service. Much of the literature demonstrates that there are frequently multiple feasible alternatives to allocate resources, and each alternative has a unique performance profile with characteristic statistics, especially in punctuality and robustness. The analysis of these alternatives frequently requires laborious and inconclusive modeling with simulation software, which is time consuming in both model programming and analysis run-time.

This presentation contributes to the literature a closed form function estimate of aggregate railway line delay propagation in response to a primary delay. This function may supplement or replace the application of simulation for exploration of alternatives. This formulation is closed form under a set of timetable-structure assumptions that are later shown, using microsimulation, to be robust to deviation from the assumptions. The formulation is derived from a finite series of deviations from the service plan (secondary delays) caused by a singular initial disruption (primary delay), and it is shown that if the initial disruption is below a given threshold (full recovery is possible), the total service plan disruption is a third degree polynomial of the initial disruption. If the total disruption is measured over less than the full recovery region, the total service plan disruption can either be linear or a second degree polynomial of the initial disruption. Further, the function may be inverted and the maximum sustainable disruption estimated by the characteristics of the line and the service plan. The results may be used to establish bounds of expected performance for simulation models, and possibly reduce the use of simulation models in some applications.

A FINITE SERIES MODEL OF DELAY IN TWO DIMENSIONS

This model proposes a closed form function that calculates the total cumulative deviation (delay, there is no earliness allowed) from the service timetable at all measurement points, as a function of timetable supplement, timetable buffer, and a single initial delay to one train. This paper suggests that cumulative deviation is appropriate for estimating the utility loss to passengers in the system under relaxed assumptions of uniformity of traffic. In the following section this model will be shown to be a reasonable guide even when the initial conditions of uniformity are violated.

This model has a two dimensional analysis horizon or domain. Many of the prior cited papers define the analysis horizon in terms of the length of line or the number of train path segments. This model specifically includes the secondary delays to trains, and thus the second dimension of the analysis horizon is the number of trains included in the cumulative delay statistic. This model will consider trains on a single line with a single direction of movement, which is a fairly common operating plan in Europe and urban North America. The time horizon of the model then begins with the train and location of the primary delay, and ends with the return of the last train to schedule within the allowed service parameter (delay threshold).

The function for cumulative delay is shown in Equation 1, for the general case, where p is the primary delay magnitude, s is the number of stations, i is the number of trains scheduled, a is the timetable supplement, and b is the timetable buffer. If the ceiling function is relaxed (which will be shown in the presentation to be appropriate), the summation simplifies to Equation 2. It will be shown in the presentation that a=b is desirable, and leads to a response surface as shown in Figure 1.

Eq. 1

$$\Gamma = \sum_{\substack{i \in \left\{1,2,\ldots,\left\lfloor \frac{p+a-\delta}{b} - s\frac{a}{b} \right\rfloor + 1\right\} \\ s \in \left\{1,2,\ldots \left\lfloor \frac{p-\delta}{a} \right\rfloor + 1\right\}}} p + a + b - sa - ib$$

Eq. 2

$$\Gamma = \frac{p^3}{6ab} + \frac{3(a+b)p^2}{12ab} + \frac{(a^2 + 3ab + 6b\delta - 6\delta^2)p}{12ab} + \frac{-a^2\delta + 9ab\delta - 3a\delta^2 - 9b\delta^2 + 4\delta^3}{12ab}$$

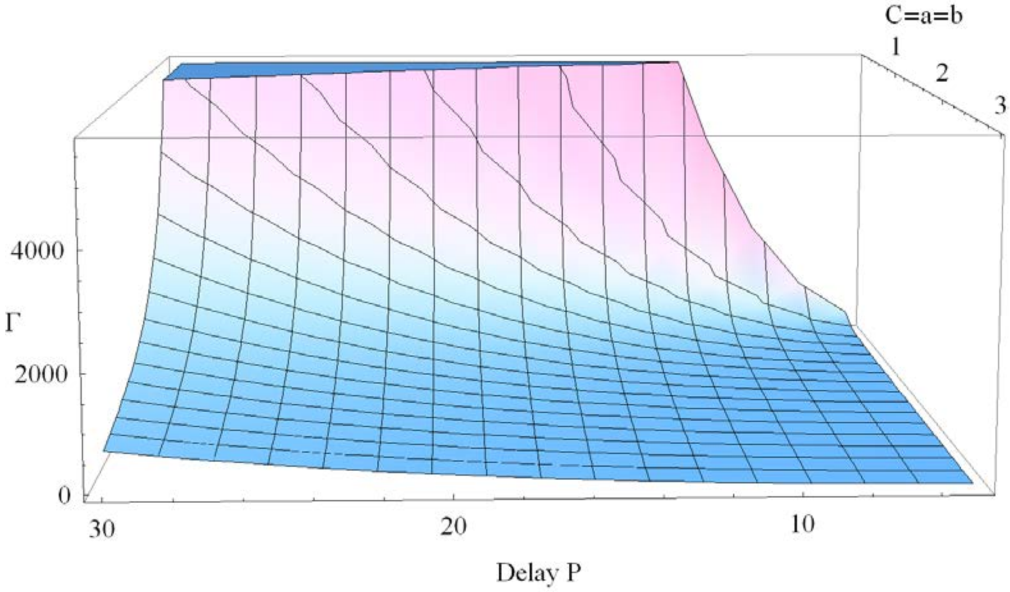

Figure 1

Case Study: Hillerød Sbane

In this section a contemporary suburban railway in Denmark is simulated and comparisons are made between the measured and theoretical system delay. The simulation is performed in OpenTrack (Nash & Huerlimann, 2004). The subject line is the Hillerød suburban railway on its northern segment from Hellerup to Hillerød (29 km.). On this segment there are eleven stations inclusive of the terminal, Hillerød, and the junction Hellerup. Hellerup is not the end of the line. All trains continue through Hellerup, through Copenhagen, and on to destinations much further south of Copenhagen.

Two simulation analyses are presented: primary delays experienced by the A service and primary delays experienced by the E service. In each case, primary delays are simulated at the Hellerup station from a uniform distribution of [0,600] seconds, and 100 replications are sampled. Only northbound traffic to Hillerød is studied. Cumulative delay is measured across both services, A and E, on the line.

It should be noted that the timetable supplement, *a*, and headway buffer, b, are severely asymmetric in the case study. They are far from the recommended level of *a*=b. The average buffer for all six measures of services A and E is 240.5 seconds. The average supplement at each of ten destination stations is 43,75 seconds. The delay threshold, δ, is zero, and all delays of any magnitude are included in the cumulative delay. In spite of this, there is strong correspondence between the theoretical and numerical estimate of aggregate delay, as shown in Figure 2.

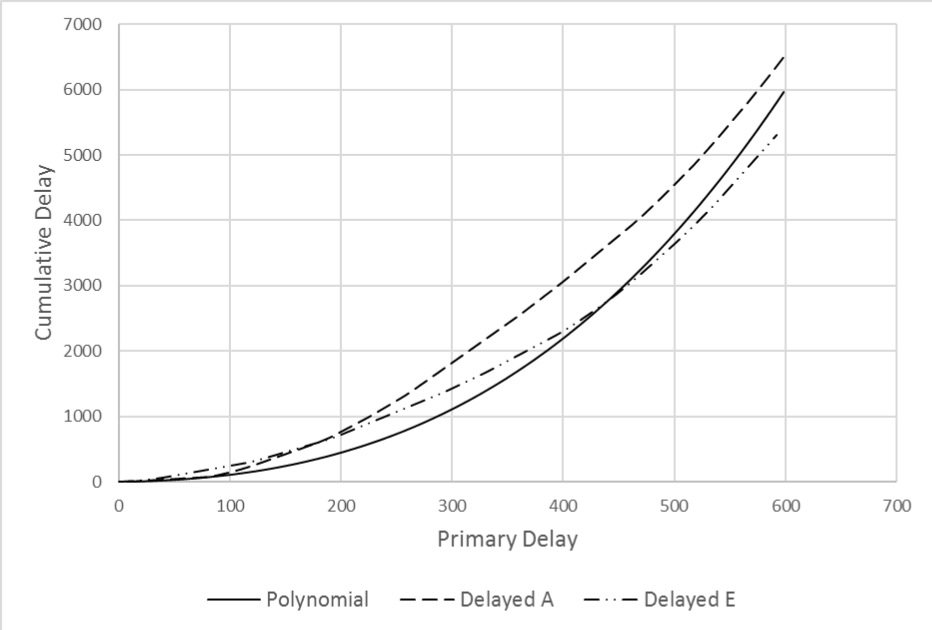

Figure 2

Conclusion

This presentation contributes to the literature a polynomial function that returns cumulative railway line delay due to a single initial primary delay. The polynomial is third degree, it is a cubic function of the primary delay, if the measurement horizon extends fully over the length of line and number of trains necessary for the disturbance to be absorbed by the timetable running time supplements and the headway buffers (the recovery region). This agrees with the earlier findings of Hasegawa et al. (1981). This paper differs from Hasegawa in that it explicitly models the discrete summation of delays, considering three parameters: supplement, buffer, and threshold for measureable delay. This results in a polynomial function of primary delay instead of the purely cubic function of delay in Hasegawa. If the measurement horizon is restricted to less than the full recovery region, the polynomial reduces to second degree and finally linear. The polynomial is an approximation of the discrete summation, and is robust over a wide range of parameters. Investigation of the contour of the polynomial finds that, in the examples considered, running time supplement and headway buffer should be equal values. Further, excessive values of running time supplement and headway buffer may result in poor timetable stability. When supplement and buffer are identical, the cumulative delay function simplifies further to a form that is easy to work with.

Simulation studies are presented where the simulated railway traffic is heterogeneous and the timetable structure deviates significantly from the ideals suggested by this analysis (the parameters are greatly asymmetric). There is a significant difference in the magnitude of cumulative delay measured using the simulation and estimated using the polynomial function, but the functional response is in agreement. Further investigation could reveal whether this is a simulation calibration issue or the result of the extreme asymmetry of the experimental model.

The average timetable supplement and buffer time can also be computed by inverting the total delay formulation, due to the close-form expression of the model. This means that given a desired punctuality and stability of service, the necessary timetable supplement may be estimated from this function. This would also ease the planning process compared to a try-and-correct approach using microsimulation. The estimated values would provide an advanced starting point for timetable planning and validation by simulation.

The analysis of the contour of the polynomial offers opportunities for further research in the proper selection of running time supplement and headway buffer. A greater variety of experimental railway lines should be simulated to confirm the result of selecting equal timetable supplements and headway buffers, and to estimate the impact of traffic heterogeneity on the polynomial assumptions. Further development of this function should consider the summation and interaction of multiple primary delays at different locations.