Optimization of timetable supplement from a passenger based socio-economic point of view

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Abstract
This article discusses how to optimize the timetable supplement in timetables. The focus of this article will be on railways, but the principle will in theory apply to all transportation modes within the area of public transportation.

When constructing timetables it is important to plan the right amount of timetable supplement. Too little timetable supplement will result in many delays, while too high timetable supplement will result in a (too) high planned travel time which will affect every departure whether or not the train is delayed. At present timetable supplement is chosen based on experience or estimates. Through a mathematical optimization it is possible to find the optimal timetable supplement. A way to do so is by using the passenger delay model in a socio-economic analysis as done by (Thorhauge & Piester, 2010).

A case study of an upgrade of Sydbanen between Ringsted and Rødby has been conducted using the passenger delay model and the methods are described in this article. The case study has shown that the optimum timetable supplement is between 6-9 % depending on the scenario. By optimizing the timetable supplement it is possible to achieve a surplus of 250-500 mio. DKK during the evaluation period compared to the proposed timetable by the Danish Transport Authority (Trafikstyrelsen, 2008). Note however that none of the investigated scenarios are socio-economic viable even though the timetable and timetable supplement is optimized.

Note that this paper is regarded as a sequel to the article “The usability of passenger delay models in socio-economic analysis” (Thorhauge, 2010). This article is based on the results of (Thorhauge & Piester, 2010).

Keywords: Timetable supplement, train delays, optimization, socio-economic, upgrade of Sydbanen
Introduction
This article discusses how to optimize the timetable supplement in timetables for railways. Timetable supplement is necessary in order to insure that the train will be on time in case of minor delays. This is important because delays have a higher value of time than (planned) driving time according to the socio-economic unit prices published by Modelcenteret (Modelcenter, 2009). On the other hand timetable supplement should not be unrealistically large, since timetable supplement will increase the planned travel time, meaning that – in the case of no (or very small) delays - the passenger will spend more time on a journey than necessary. Therefore it is crucial to incorporate the “right amount” of timetable supplements when planning timetable. However the right amount of timetable supplement varies from timetable to timetable. So how to find the right amount of timetable supplement? A method using the passenger delay model combined with a socio-economic analysis in order to optimize the timetable supplement is discusses in this article.

Principles and calculations
The following chapter will discuss and describe the method used to find the optimal timetable supplement. The overall hypothesis is that it is possible to increase the timetable (and thereby decrease delays) to a certain point, from which it is not socio-economic viable to increase the timetable supplement any further, due to longer scheduled travel times for every departure. This point can be considered as the most socio-economic viable timetable supplement, and can be described as “equilibrium” between the amount of delays and the schedule travel time. (Landex, 2008) illustrates the principle as shown below:

![Figure 1: The hypothesis that it is possible to optimize the timetable supplement so that the socio-economic surplus cannot improve any further. Source: (Landex, 2008).](image)

Determining the optimal timetable supplement can be a difficult task however. The optimal timetable supplement depends on several factors; the overall available capacity of the railway line, the capacity consumption, the homogeneity of the timetable, the trains driving characteristic, etc.
Assume an example with a timetable containing three trains. The minimum (physical) driving time from A to B is assumed to be 10 minutes, and the three trains are assumed to experience the following three driving times in the realized timetable (due to delays):

- Train 1 uses 10,2 minutes to drive from A to B.
- Train 2 uses 10,3 minutes to drive from A to B.
- Train 3 uses 10,5 minutes to drive from A to B.

Furthermore, for simplification it is assumed that the three trains will have a total of one passenger each (although this is obviously not going to be economically feasible) and that these passengers only experience driving time – and in some cases delays\(^1\). The value of driving time is assumed to be 100 DKK/hour, while the value of time for delays is assumed to be the double (according to (Transport- og Energiministeriet, 2004) and (Modelcenter, 2009)), namely 200 DKK/hour. With these assumed simplifications the table below shows how to calculate the optimal timetable supplement as equilibrium between the socio-economic cost of driving time and the socio-economic cost of delays according to different timetables supplements. The optimum timetable supplement is marked within the red box.

![Table 1: Simple example of how to calculate the (relative) socio-economic surplus as a function of the timetable supplement.](image)

The method used to calculate the socio-economic surplus is to calculate the total socio-economic cost of all the investigated timetable supplements (as done in the row marked with blue). This is possible since the time spend on transportation is considered a cost (or in other words; as “lost time” for the passenger and/or the society). In order to calculate the total socio-economic cost the cost for driving time and the cost of delays must be calculated. The total cost of driving time is calculated as\(^2\):

\[
\text{Total cost, driving time, timetable supplement}_i = \sum_{\text{Train } i=1}^{\text{Train } j} SDT A \to B_{\text{train } i,j} \cdot \text{no. of pass. } A \to B_{\text{train } i,j} \cdot \text{unit price driving } t
\]

The total cost of delays is calculated as:

\(^1\) Waiting time, etc. are ignored for simplification.

\(^2\) Abbreviations in equation are as follows: SDT = Scheduled Driving Time, RDT = Realized Driving Time.
Total cost, delays, timetable supplement $j$

$$
= \sum_{i=1}^{Train} \left( \text{RDT } A \rightarrow B_{\text{train } i,j} - \text{SDT } A \rightarrow B_{\text{train } i,j} \right) \cdot \text{no. of pass. } A \rightarrow B_{\text{train } i,j} \cdot \text{unit price}_{\text{delay}}
$$  \hfill (Formula 2)

In order to calculate the socio-economic surplus, the timetable supplement with the highest total socio-economic cost must be identified. The socio-economic surplus is calculated as the difference between the total socio-economic cost of the timetable supplement which has the highest overall socio-economic cost (in this example a timetable supplement of 0 % is used) and the socio-economic cost for a given timetable supplement. The formula is listed below:

$$
\text{Socio economic surplus } j = \max(\text{socio economic cost } j) - \text{Socio economic cost } j
$$  \hfill (Formula 3)

The figure below illustrates the principles of how to convert the socio-economic cost to a relative socio-economic surplus, as done with the formula 3. Note that the socio-economic cost of the scenario with a timetable supplement of 0% is the highest among the investigated scenarios. Therefore this scenario will be the scenario of reference from which the socio-economic gain from all the other investigated scenarios will be calculated. In other words; this means that a timetable supplement of 3 % is 1,3 mio. DKK more socio-economic feasible than a timetable supplement of 0 % (which is considered as the least socio-economic feasible scenario).

![Optimum timetable supplement - example](image)

**Figure 2: Graph illustration the conversion from socio-economic cost (red) to socio-economic surplus (blue).**

The way of calculating the socio-economic surplus means that the surplus found in this ways is relative to the scenarios investigated. This is due to the fact that the surplus is calculated from the timetables supplement with the least feasible socio-economic value. Therefore the socio-economic surplus
calculated in this example cannot directly be used in comparison with similar analysis conduction on other scenarios. In order to compare different timetable (as well as different infrastructure upgrades) three methods of analysis have been developed with this example being the most simple of the three methods of analysis (described in the next chapter).

Methods of analysis

In order to conduct different analysis three methods have been developed. The three methods increase in complexity, with method 1 being the most simple and method 3 the most complex. The example above is in theory (although unrealistic) an example of the most simple analysis, method of analysis 1, which can be used only to calculate the optimum timetable supplement. In the example above all time components except driving time and delayed time is assumed to be none-existing, which in real life situations rarely will be the case. The three methods of analysis (and their limitations) are described below:

- Method of analysis 1 can be used to find the optimum timetable supplement for a given timetable. This is useful in order to optimize a given timetable without changing the overall structure of the timetable. Method of analysis 1 however cannot be used to compare two (or more) different timetables, because the gain is relative and not absolute. To compare timetables method of analysis 2 or 3 must be used.

- Method of analysis 2 includes the everyday cost of using the infrastructure and the trains, and can therefore be used to compare different timetables to find the timetable and timetable supplement with the overall highest socio-economic surplus. This is useful when deciding on the structure of the timetable (i.e. homogeneous vs. heterogeneous timetable, number of trains, etc) as well as the optimal timetable supplement for that specific timetable. The limitation of method of analysis 2 is that all timetables must use the same infrastructure.

- Methods of analysis 3 include cost of upgrading the infrastructure and works with an evaluation period making it possible to see the gains or cost in the future. This method can therefore be used to compare different timetables where the infrastructure, timetable, and timetable supplement are different in order to find the socio-economic optimal scenario. This is for example useful in order to investigate whether an infrastructure upgrade (and thereby a new timetable) is socio-economic feasible. It is possible to investigate several timetables on the upgraded infrastructure and compare these with the base-scenario and find the optimum timetable supplement of each of the investigated timetable in order to find the overall most socio-economic feasible timetable and timetable supplement.

The figure below summarizes the capabilities and limitations of the three methods of analysis:

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3 The other components are elements such as waiting time, first waiting time, hidden waiting time etc.
Table 2: Overview of capabilities of the three methods of analysis.

The following figure shows which effects should be included in the socio-economic analysis in the above three methods of analysis. The overall principle is that when using method 1 all passengers related effects should be included. When using method 2 all passenger and train related effects should be included. Finally when using method 3 all socio-economic effects should be included.

Table 3: Overview of socio-economic effects in methods of analysis 1-3.

In this article the main focus will be on method of analysis 3. Note that method of analysis 1 and 2 can be considered as simplifications of method of analysis 3. The overall calculation principles are identical among the three methods of analysis.

A case study: Sydbanen

The following chapter will describe the principle described previously in Methods of analysis applied to a specific case study; an upgrade of Sydbanen between Ringsted and Rødby. An upgrade of Sydbanen is interesting in accordance with the coming Fehmarn-connection, see more (www.fehmarnlink.com, 2010). In this case Sydbanen is assumed to be upgraded to a double railway line and prepared for speeds of 160 km/h (Thorhauge & Piester, 2010). A total of four different variants of the upgraded scenario are investigated as illustrated below. Method of analysis 3 will be applied in this case since different infrastructures will be compared.
The scenarios are tested against a base-scenario (do-nothing-scenario). The socio-economic gain as a function of the timetable supplement is shown in figure below:

![Socio-economic development over 50 years compared to the base-scenario](image)

First of all it must be noted that none of the investigated scenarios are socio-economic feasible compared to a do-nothing-scenario. This is due to high construction costs of the investigated scenarios. However the figure shows two tendencies; 1) a double railway line across Storstrøm is more socio-

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4 Around 8-9 billion DKK depending on the specific scenario – based on estimates from (Trafikministeriet, 2008) and (COWI, 1999).
economic viable than a single railway line across Storstrøm and 2) the timetable with many trains seems as the better of the two investigated timetables from a socio-economic point of view (although the operation economy might show otherwise). Note that, the figure show a linear dashed line and a dotted parabola. The linear line shows the socio-economic value of the timetable proposed by the Danish Ministry of Transport (Trafikstyrelsen, 2008)\(^5\) while the dotted line is a regression of the parabola, which will provide information about the “theoretical” optimum\(^6\). From the equation of the parabola it is possible to calculate the top-point of the parabola, and thereby calculate the optimal timetable supplement. Doing so for the four different parabolas gives the following results:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Optimum timetable supplement</th>
<th>Annual gain of optimum timetable supplement compared with the proposed timetable</th>
<th>Total gain after an evaluation period of 50 year compared with the proposed timetable</th>
<th>Total gain after an evaluation period of 50 year compared with the do-nothing-scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>6.9 %</td>
<td>14 mio. DKK</td>
<td>254 mio. DKK</td>
<td>-2644 mio. DKK</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>6.7 %</td>
<td>17 mio. DKK</td>
<td>310 mio. DKK</td>
<td>-2524 mio. DKK</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>8.3 %</td>
<td>18 mio. DKK</td>
<td>328 mio. DKK</td>
<td>-1695 mio. DKK</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>7.6 %</td>
<td>26 mio. DKK</td>
<td>482 mio. DKK</td>
<td>-1184 mio. DKK</td>
</tr>
</tbody>
</table>

Table 5: Optimum timetable supplement and the socio-economic surplus.

The table shows that the optimal timetable supplement is between 6-9 %. Furthermore the table shows that it is possible to optimize the timetable supplement compared with the proposed timetable with an annual gain of 14-26 mio. DKK, which will transform to a surplus of approximately 250-500 mio. DKK during the period of evaluation. Last but not least the table shows that it is not socio-economic viable to upgrade Sydbanen compared with the existing infrastructure (do-nothing-scenario) even though the timetable supplement of the proposed timetables is optimized.

**Weaknesses**

The methods developed and described in this paper are useful in order to optimize infrastructure, timetables and timetable supplement of the Danish railways. However there are some weaknesses with the method which are describe below:

- **Knowledge of (future) realized travel times is necessary in order to determine the train delays:**
  If the method is used to evaluate existing railway lines, this is not going to be a problem. On the other hand, if the method is used in order to predict whether a change in the infrastructure,  

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\(^5\) Note that, the structure of the two different timetables are identical – the only difference is that timetable A has some line variants which only runs in rush hour oppose to timetable B where they all run all day.

\(^6\) It is useful to use the theoretical optimum in order to avoid the fluctuations. The fluctuations occur due to simplification in the model process where number of launch in the passenger delay model is set equivalent with the number off minutes – meaning that the model will have a launch per minute. Although this number is fairly high (often a launch per 10 minutes is used), it is not detailed enough to avoid large fluctuations as seen in the graph (small fluctuation could be expected). It is possible to decrease these fluctuations by increasing the number of launch to approximately a launch per second, but keep in mind that this will increase the computational power needed as well as the time consumption.
timetable and/or timetable supplement is socio-economic viable, knowledge of the delays of the future railway line is necessary. Obviously this information is not available, so in order to compensate for the lack of knowledge the future delays must be estimated (in this case by the use of the simulation software RailSys). However estimating future delays is a time consuming and difficult task, which can cause biased results.

- **Lack of a “solve to optimum”-function**: The problem is very well demonstrated with the small example in the beginning of this article. The main problem is that it is necessary to manually calculate each of the scenarios in order to compare different timetable supplements. In the example timetable supplement of 0 %, 1 %, 2 %, 3 %, 4 %, 5 % and 6 % were calculated and the timetable supplement of 3 % seems to be the optimum timetable supplements among the investigated scenarios. However in theory a timetable supplement of (for example) 3.1% or 2.9% can prove to be the overall optimum timetable supplement. One way to overcome this problem is to calculate the regression to the polynomial of second order, as done in the case of Sydbanen. In the end it all comes down to the number of the investigated scenarios as well as the number of launch for each scenario. Of course many scenarios (with a large amount of launch) will increase the accuracy of the results. The downside is that the workload and time consumption increases very rapidly.

- **The passenger delay model is developed to the suburban railway network, S-trains, in Copenhagen**: The fact that the passenger delay model originally is developed for the S-train means that model data\(^7\) only are available for the S-train network. It is not impossible to obtain model data for other railway lines, but these have to be imported and converted to a specific format and in worst case made manually. The task of creating new model data can quickly prove to be a time consuming task, especially when taking the debugging-aspect into consideration as well.

### Perspectives

The methods described in this article can be used for optimization of infrastructure, timetables and/or timetable supplement from a socio-economic point of view. However the main hurdle is the creation of model data and conversion of timetables. If data is created the method of analysis 2 can be used to optimize the entire Danish railway network without even upgrading the infrastructure at all. Especially DSB S-tog can use the method described in this article to optimize the suburban railway lines in the greater Copenhagen area since all the model data and timetable already exists. The method of analysis 3 can be used to predict whether or not a project is viable from a socio-economic point of view when modeling with delays – as illustrated by the example of an upgrade of Sydbanen.

The long term vision is that the model and method is expanded to take other (public) transportation modes\(^8\) into account as well, and thereby optimize the complete public transportation network for the entire greater Copenhagen region as a coherent public transportation network.

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\(^7\) Station-nodes, links, timetables, modelparameter, etc.

\(^8\) Busses, metros, regional trains, harbour busses and maybe future light rails.
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