Shape Optimizing of a Sleeping Policeman

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1. Introduction

The number of speed bumps (”sleeping policemen”) have in many countries increased steadily in the last decade. They are commonly used in the smaller streets near peoples homes where it is important that the speed of the cars is kept very low (e.g. 30 km/h), primarily because of the presence of children unaware of the traffic laws. Many drivers have the feeling that with increasing speed the ride over the bump becomes more pleasant, this observation is correct and is mainly due to an unsatisfactory design of the shape of the bump. The results will show that with small changes it is possible to improve considerably the response to the driver.

The shape of a sleeping policeman is optimized with respect to the response characteristic of a car going over the bump. The objective is that the ride is as pleasant as possible when crossing the bump below the maximum allowed speed, while being unpleasant when the driver is going too fast. The shape of the bump is described by a number of amplitudes of some basic functions, that are orthogonal in the sense that each function contributes something new to the design space. Optimization is performed with numerical sensitivities, from a 2D multibody system simulation, and the results show that it is possible to achieve great improvements in the bump design. The optimization method is not specialized to a specific mechanism and may be used to treat other multibody systems to change the response characteristics.

2. Modelling

To optimize the shape of a sleeping policeman or speed bump we have to be able to analyse the problem, and therefore to model the problem. The modelling falls in two parts; the modelling of the car and the shape representation of the bump. The assumptions that the problem can be modelled in 2D is used. The shape representation is done with the eigenvectors from elementary cases for uniform beams (see figure 1). These functions are orthogonal in the sense that each function will contribute something new to the design. The functions used can be found in many books e.g. in Volterra and Zachmanoglou (1965). In this book the case of one end fixed and the other end having fixed rotation but being free to translate is however not found. These functions, hereafter called fixed-sliding, are given by

$$X_n(x) = \cosh(k_n x) - \cos(k_n x) - \frac{\cos(k_n L) - \cosh(k_n L)}{\sin(k_n L) + \sinh(k_n L)} \left(\sinh(k_n x) - \sin(k_n x)\right) \qquad (1)$$

where

L is the length of the domain

x length parameter $0 \le x \le L$

k is the solution to the transcendent equation $\sin(kL)\cosh(kL) + \cos(kL)\sinh(kL) = 0$

n is the solution number

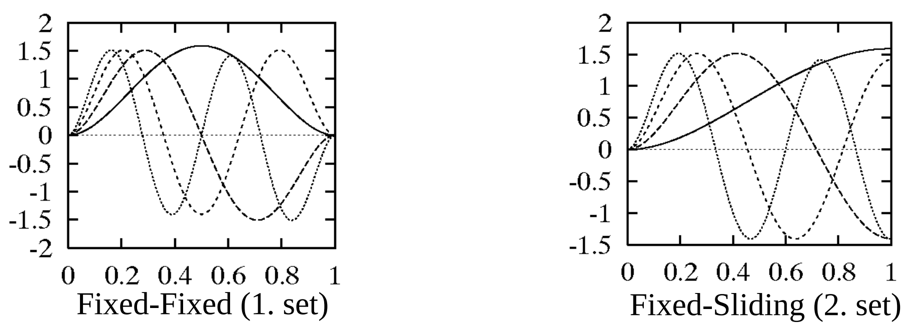

Figure 1. Functions used to describe the speed bump.

In the modelling of the car we use the two different car models shown in figure 2, The choice is to use the generic car as the standard or reference car, and verify with the jeep that the result of the optimization for the standard car do not give an unsatisfactory ride for a car design far from the one used in the optimization.

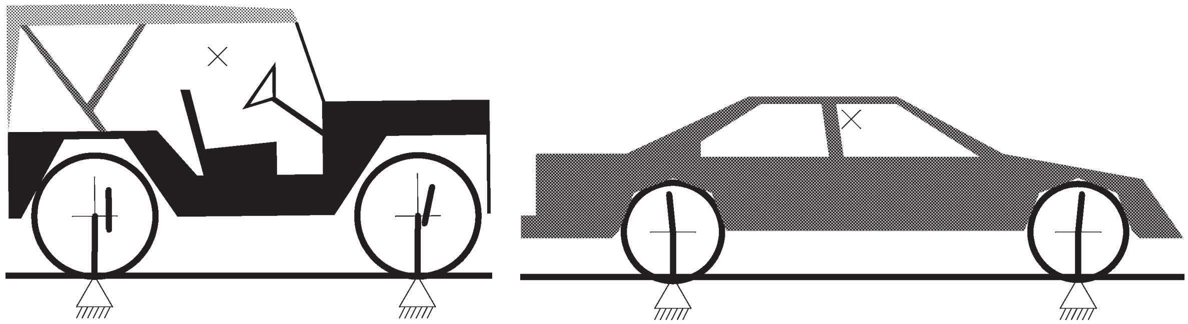

Figure 2. Schematic drawing of the jeep (sports utility vehicle; SUV) and the generic car used.

3. Optimization

The parameter used in this study to describe the level of comfort is the maximum acceleration that the head of the driver will experience in a ride over a bump

$$\Gamma_c = Max\left(\sqrt{a_x^2(t)+a_y^2(t)}\right) \qquad t \in \left[t_{start\ \ ride} : t_{end\ \ ride}\right] \tag{2}$$

The objective of the optimization is given by

$$Minimise\ \left(Max\ \left(\left(a_{\max}(\dot{x})\right)_{desired} - \left(a_{\max}(\dot{x})\right)_{response}\right)\right) \qquad \dot{x} \in [0:100]km/h \tag{3}$$

where

$\dot{x}$	the initial speed of the car going over the bump
$\left(a_{\max}(\dot{x})\right)_{desired}$	desired curve of the response
$\left(a_{\max}(\dot{x})\right)_{response}$	response curve of actual bump and car pair

To carry out the optimization the objective is discretized and we use a Taylor expansion to fit the problem to a Linear Programming problem. Sequential Linear Programming (SLP) is used as the overall optimization method. For more details on the optimization and the modelling see Pedersen (1998).

4. Results

The results from five different optimizations are given, the first two examples are non-symmetric bumps while examples 3 through 5 are symmetric. In all of the examples the optimization is done with the generic sedan and a bump length of 4 meter. The choice of the length is a compromise; if the bump is longer we can get a better response curve but because of the cost of manufacture the length should be limited.

The first example is an optimization with the design speed of 30 km/h. The optimization is done with eight design parameters from the 1. set (figure 1). The initial bump for the optimization corresponds to an amplitude of 0.01 for the first function of 1. set. The response curve of the generic car and the utility vehicle is shown in figure 3. The shape of the bump is shown in figure 4, and the amplitude values are given in table I.

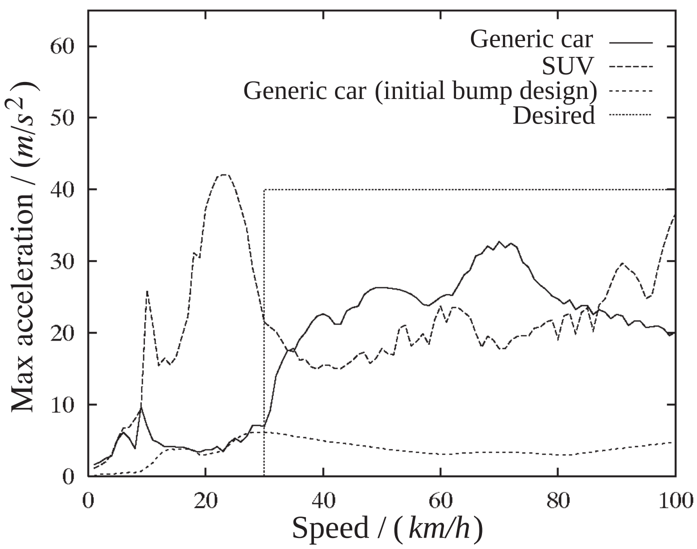

Figure 3: Response curves for generic car and SUV going over bump (non-symmetric, 4 meters, 30 km/h).

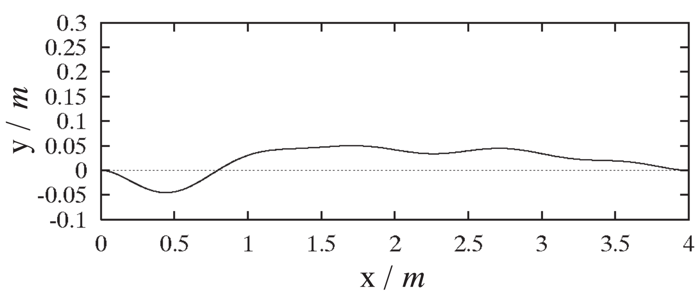

Figure 4: Shape of optimized bump (non-symmetric, 4 meters, 30 km/h).

1. set	1	2	3	4
	$7.3472\ 10^{-3}$	$-1.0969\ 10^{-3}$	$-5.4042\ 10^{-4}$	$-2.8044\ 10^{-3}$
	5	6	7	8
	$-2.3116\ 10^{-3}$	$-1.5750\ 10^{-3}$	$-9.1714\ 10^{-4}$	$-1.3114\ 10^{-3}$

Table I: Amplitude values of optimized design (non-symmetric, 4 meters, 30 km/h).

The results show a great advantage in the response curve for the generic car; the maximum accelerations are kept at an acceptably low level when the speed of the car is below the design speed of the bump, while rising to an unpleasant level when the speed limit is violated. The ride of the utility vehicle over the same bump results in a less advantageous curve; the accelerations below the design speed are too high. This is expected because the SUV is much stiffer than the generic car. In the design of the bump it is most important how the response is in the range $[10{:}80]$ *km/h* because it is in this range that most speeds are experienced; we will therefore neglect the response curve outside this domain.

The second example is the same optimization as the first example, but this time the design speed of the bump is changed to 40 km/h. The results of the optimization are shown in figures 5 and 6, together with table II.

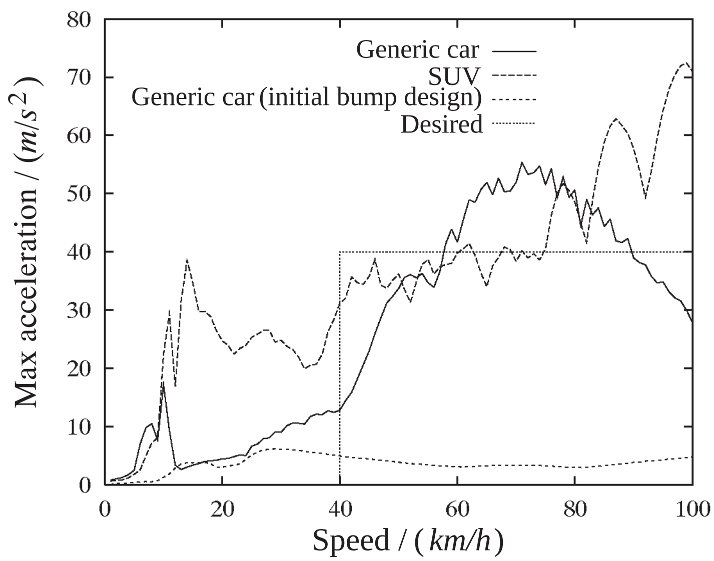

Figure 5: Response curves for generic car and SUV going over bump (non-symmetric, 4 meters, 40 km/h).

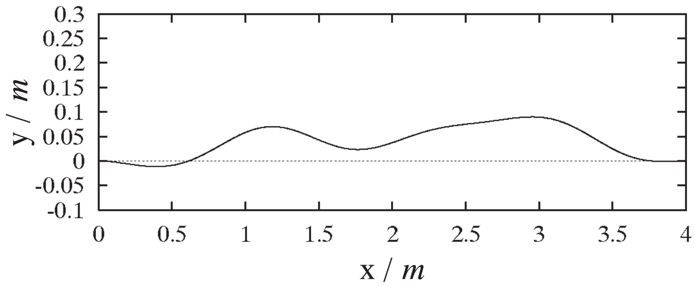

Figure 6: Shape of optimized bump (non-symmetric, 4 meters, 40 km/h).

1. set	1	2	3	4
	$1.1064\,10^{-2}$	$-3.3330\,10^{-3}$	$4.2963\,10^{-3}$	$-6.3945\,10^{-4}$
	5	6	7	8
	$-2.7562\,10^{-3}$	$-1.3778\,10^{-3}$	$-1.1857\,10^{-3}$	$9.7744\,10^{-4}$

Table II: Amplitude values of optimized design (non_symmetric, 4 meters, 40 km/h).

The results show that we are able to achieve the desired change in the response curve, and it is noted that the shape is able to adjust to the new desired curve. The response curve of the utility vehicle is again less advantageous. One of the main results is that the shape of the bump has changed considerably from the design of the first example. The only design difference between speed bumps currently in use seems to be the height and length; there is no significant shape difference. The results here indicate that the shape of the bumps should be different, depending on the speed limit. In examples 3 and 4 the optimizations from examples 1 and 2 are repeated but this time for a symmetric bump. In the two examples the optimization is carried out with eight design parameters; 4 from the 1. set and 4 from 2. The results are shown in figures 7-10 and tables III-IV.

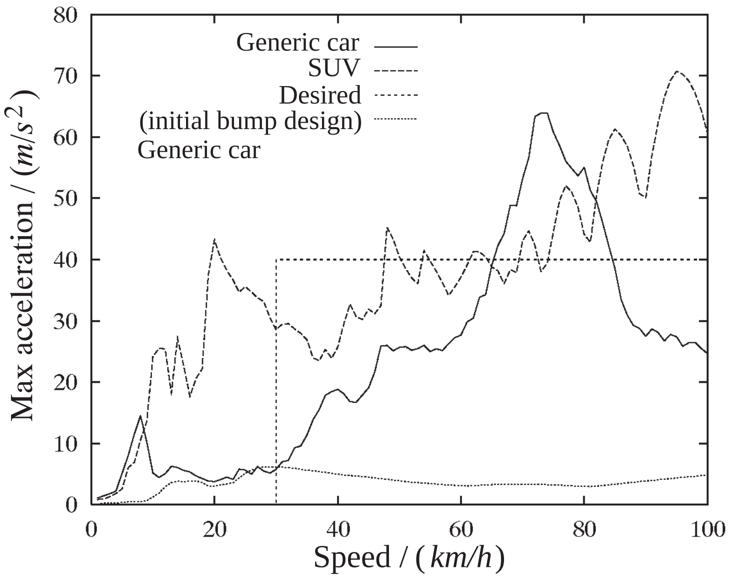

Figure 7: Response curves for generic car and SUV going over bump (4 meters, 30 km/h).

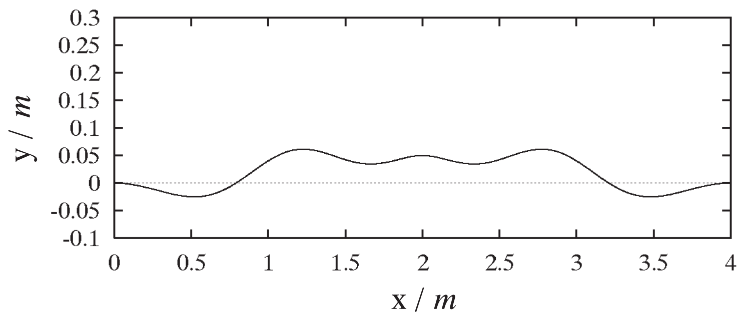

Figure 8: Shape of optimized bump (4 meters, 30 km/h).

	1	2	3	4
1. set	$-3.3754\,10^{-3}$	$-2.2331\,10^{-3}$	$-4.2688\,10^{-3}$	$2.9190\,10^{-3}$
2. set	$1.8707\,10^{-2}$	$-1.9015\,10^{-5}$	$-4.3134\,10^{-3}$	$-7.6120\,10^{-4}$

Table III: Amplitude values of optimized design (4 meters, 30 km/h).

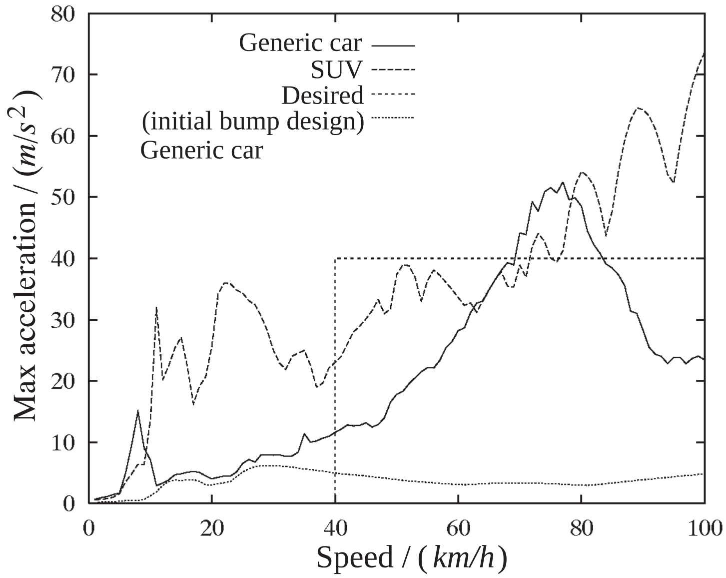

Figure 9: Response curves for generic car and SUV going over bump (4 meters, 40 km/h).

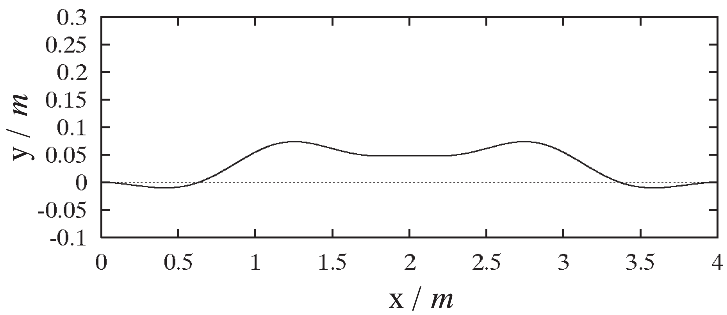

Figure 10: Shape of optimized bump (4 meters, 40 km/h).

	1	2	3	4
1. set	$3.8876\,10^{-3}$	$-4.1626\,10^{-3}$	$-2.4955\,10^{-3}$	$1.4228\,10^{-3}$
2. set	$1.7462\,10^{-2}$	$4.4881\,10^{-4}$	$-2.5473\,10^{-3}$	$-4.7010\,10^{-4}$

Table IV: Amplitude values of optimized design (4 meters, 40 km/h).

We see the same basic result in examples 3 and 4 as in the first two examples, a great advantage in the response curve for the generic car is achieved while the ride of the utility vehicle over the same bump results in a less advantageous curve. The results also show that the shape is able to adjust to the new desired curve. The main difference between the examples is that the results in examples 3 and 4 is less advantageous compared two examples 1 and 2, this seems natural because we have restricted the optimization considerably.

In the first four examples, the shape of the optimized bump was below the street level at some point. From a manufacturing point of view this may not be a desirable result, and the final example is therefore the same optimization as the third example, but this time the restriction that the shape of the bump most not have negative y-values is imposed. The results of the optimization are shown in figures 11 and 12, together with table V.

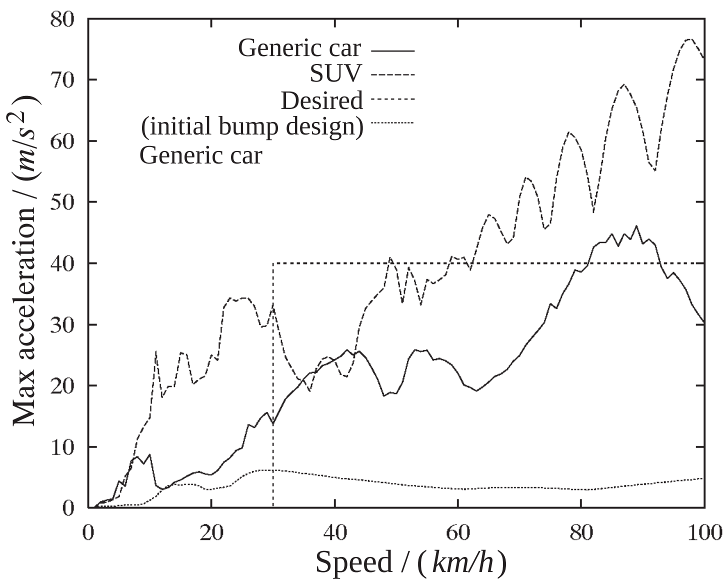

Figure 11: Response curves for generic car and SUV going over bump (4 meters, 30 km/h).

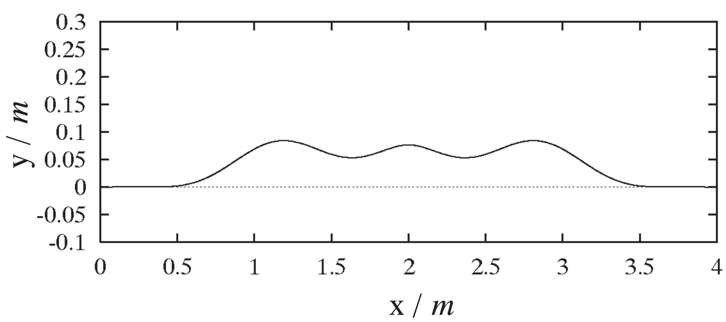

Figure 12: Shape of optimized bump (4 meters, 30 km/h).

	1	2	3	4
1. set	$4.3617\ 10^{-3}$	$-6.3064\ 10^{-4}$	$-4.1525\ 10^{-3}$	$3.5724\ 10^{-3}$
2. set	$2.3588\ 10^{-2}$	$-8.5325\ 10^{-4}$	$-1.6834\ 10^{-3}$	$-1.1657\ 10^{-3}$

Table V: Amplitude values of optimized design (4 meters, 30 km/h).

5. Conclusion

It is shown how the response characteristics of a car going over a speed bump can be optimized. The maximum acceleration to the head of the driver when crossing the bump at a speed below the speed limit of the bump (the design speed of the bump) is minimized, while the driver will experience an unpleasant acceleration when exceeding the speed limit. Optimized results are shown for a 4 meter long bump with design speeds of 30 km/h and 40 km/h, using an average car (generic sedan). The resulting response curves of the optimized bump design are shown for the generic sedan and a sports utility vehicle; the SUV design is considered to be too far from the average car and is used to show that the response curve for this car model does not result in unsatisfactory results. The objective used to quantify the comfort of the driver is the peak acceleration to the driver's head as the bump is passed. The modelling is done in 2D, so we may use 2D multibody simulation, with the reduction in CPU-time that follows from this. The shape of the bump was described by amplitudes of global functions. The functions used to describe the shape are used because they are able to describe a large variety of shapes with only a few parametres.

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