Forecasting developments of the Car fleet in the Altrans model

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1. Introduction

The Altrans (ALternative TRANSport systems) model has during the past few years been developed at DMU. The work presented in this paper is a small part of the overall model complex, and has its main importance in the forthcoming development of Altrans into a forecasting instrument. The Altrans model complex comprises of several submodels. The general outline of the model will be described shortly in the following section. The purpose of this paper is not to give a detailed description of the overall model, but to describe a small submodel calculating the composition of the car fleet determined in the central Altrans behavioural model. A more thorough description of the central parts of the model can be found in forthcoming reports from NERI (Christensen, 1999 and Rich, 1999).

As the main purpose of the developed model is to be able to forecast energy consumption and emissions from person transport, the model is estimated using time series data. The basic data for implementing time series estimation is detailed information of the stock of cars in Denmark in the period from 1991 to 1997 augmented by more aggregated information from 1977 to 1990. The data could be even further augmented with information from 1960 and onward where the level of detail is much lower. It is therefore considered uninteresting for the work here. The main characteristics of the basic data are described in section 3. It is both important and interesting to know these characteristics, as the model should be able to replicate these, but also for identifying the key factors determining the actual development.

The development in the car stock composition is made of three different parts. The starting point is the existing stock of cars. The stock changes as a result of acquisitions of new cars and the scrapping of old cars. In section 4, the model combining these three elements is described. The composition of acquisitions is calculated by a model developed by a Danish consultancy: CowiConsult, and will not be described in further detail. The last interesting element therefore is the model calculating the scrappage. The different elements and considerations of this model are described in section 4, which also includes the estimation results of the relevant determining variables.

The penultimate section of the paper describes shortly how the use of relatively detailed information on emission factors are used in the model. These are especially important for forecasting purposes. The final section sum up the findings of the paper.

2. The Altrans model complex

The Altrans model is made of two central submodels: a geographical GIS based model, calculating in vehicle, access, egress and waiting times, journey lengths, different accessibility measures like number of working places, number of public transport km. within each zone etc. The model concept is described in Thorlacius (1998). The other main submodel is a behavioural model based on the Danish national transport survey. The behavioural model estimates the traffic performance of the individuals on each mode of transport and the total stock of cars.

The general outline of the model complex is illustrated in figure 1. The core of the model is the behavioural model, to which the geographical model is supplying input. Another equally important data input is the national travel survey giving information on the travel behaviour of approximately 12000 individuals in each of the years 1995 to 1997[1]. The information includes knowledge of several socio-economic variables supplied by the respondents (income, type of household, number of children etc.) and the different trips the individual made the day prior to the interview. The illustration is actually a slightly simplified version of the model, as the geographical and the behavioural model interacts in calculating distances etc. for the different destination alternatives the individuals face.

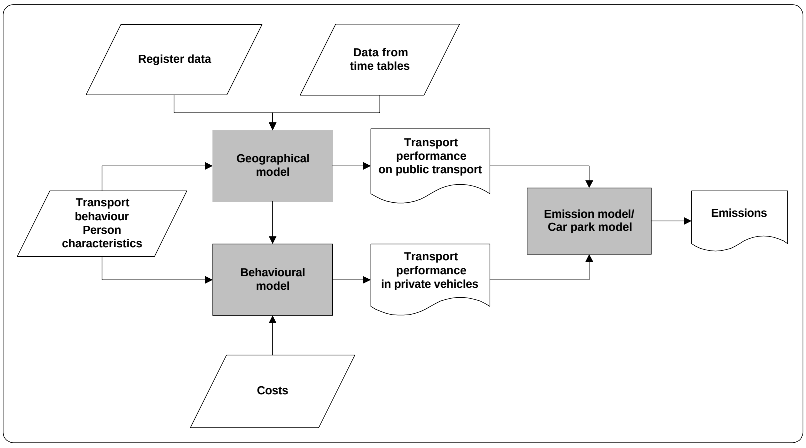

Figure 1. The ALTRANS model complex.

The individual elements in the behavioural model are described in Rich (1999). Most of these elements are of the discrete choice type, giving probabilities to different alternatives. Based on these probabilities the absolute level of traffic performance (and transport performance), and the car ownership are calculated. The way the model works is not of main interest here, and will therefore not be explored any further.

The model described in this paper is the shaded box in the right hand side of the figure.

3. Historical developments of the Danish car fleet

The data used in this project is from two different sources: a detailed database from Statistics Denmark including information on number of cars in categories defined by *model, make, type, weight, fuel type, ownership and year of first registration*. Information on this level of detail, however, is too detailed for the kind of model estimation we are interested in. This is due to the fact that information on the energy consumption of the different vehicle categories are not yet supplied in the database. It is therefore necessary to use the European database on energy consumption and emissions, Copert[2], to determine these factors. The Copert

[1] Actually the national travel survey has in its present set-up been carried out since 1993, but information vital for the model was not included until December 1994. The data from 1998 has just recently been prepared, and has therefore not been used in the present version of the model estimation. The travel survey is illustrated in figure 1 as the box including transport behaviour and person characteristics.

[2] See Ahlvik et al. (1997).

resolution on vehicle categories is limited to three different engine sizes and a series of fuel categories. As will be described in the penultimate section the age of the cars are important for the calculations. The procedure for determining the emissions based on age is also presented in that section. The implication of this is, that it is only necessary to look at vehicles in three weight categories, two fuel categories and twenty age groups.

The second important set of information come from the yearly statistics, published by the Danish car importers (De Danske Bilimportører, 1996). The statistics include information on number of cars in aggregate groups made of fuel, weight, and age categories from 1977 to 1991 with the possibility to extend the period to 1960 (the information on age, however is very limited before 1977, and also quite limited from 1977 to 1985). The two independent data sources are therefore of quite different levels of detail.

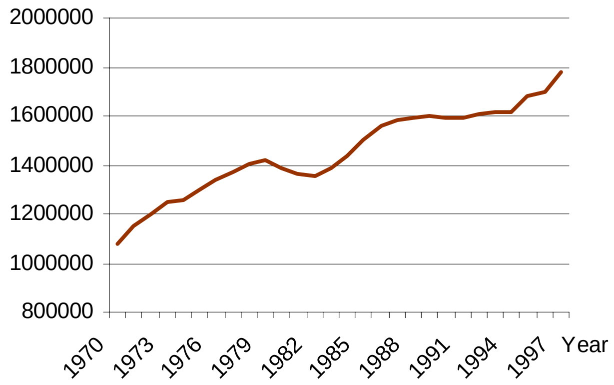

Figure 2 Stock of cars in Denmark from 1970 to 1997.

The stock of cars in Denmark has increased from approximately 1.100.000 cars in 1970 to just under 1.800.000 cars in 1997. The development is shown in *Figure 2*. The figure shows several interesting things. First the “explosive” development in personal cars in the 70’s, secondly the oil crisis around 1980. From the turning point in 1984 there were a fast increase in the number of cars, which again ended abruptly in 1987, after which the stock remained relatively unchanged for 8 years. The reason for this change in the overall trend was a series of heavy fiscal policies limiting the ability of private households to purchase durable goods, including personal cars, in combination with a general economic recession. The recession ended in the early 90’s, and since then there has been a large increase in the number of personal owned cars. The reason for these overall developments are found when looking at the acquisition and scrappage. The scrappage in the period 1985 to 1997 is illustrated in *Figure 3*.

One interesting element in the two figures is that the developments of the scrapping and the overall development seem to point in opposite directions; until 1986 there were a steep increase in the amount of scrapping, and at the same time there were an equally increase in the overall stock. Also from 1987 until mid 1990’s there were a slight decline in the number of scrapping tending to increase the overall stock. But in the same period the overall stock remained relatively unchanged. The reason for this is that the acquisition of new cars has neutralised the effect of the scrapping.

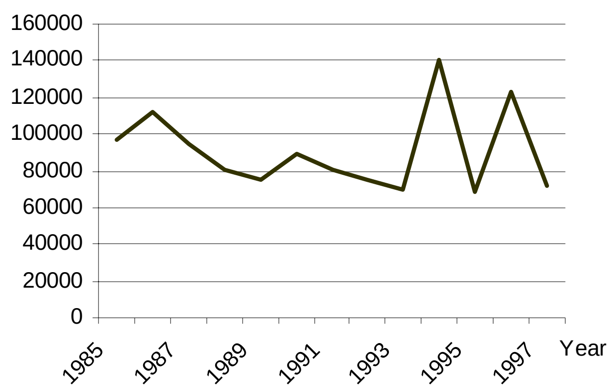

Figure 3 The scrappage of cars from 1985 to 1997.

The peek of scrappings in 1994 is

explained by a temporary subsidy scheme giving 6000 DKK to the car owners when their cars were scrapped. The reason the government introduced this premium or subsidy can also be seen in the figure. In the years preceding 1994 the scrap rate was very low resulting in a stock of old unsafe cars with high energy consumption and emissions. An analysis looking at the effect of the premium is given in Transportrådet (1995).

The peek in 1996 is harder to explain as no similar causality can be found. The only thing that we have been able to find is that a lot of cars were unregistered at the time of making up the stock and then reregistered in 1997. The reason for this could be a sudden change in some economic parameters leading especially company owned vehicles to be sold to car re-sellers, and thereby be temporarily "scrapped" at the time of making up the stock , only to return the year after. This however is only part of the explanation, but until more information is available it is impossible to come any closer to the truth.

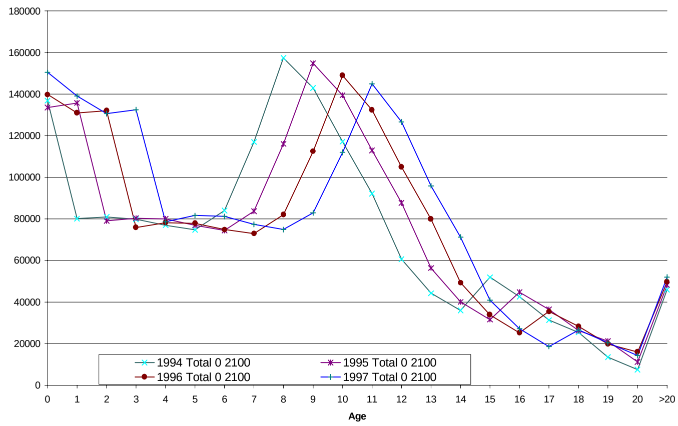

Figure 4 **Composition of car fleet on age in the years 1994 to 1997.**

At this point it is necessary to go on step deeper in analysing the development in the car stock and look at cars in different categories defined by weight, fuel and first registration year. This decomposition leads to 120 categories (made up of 2 fuel types: diesel and petrol, 3 weight categories: below 800 kilos, between 800 and 1000 kilos and above 1000 kilos[3], and finally 20 one-year age groups). An interesting point here is that cars of different age faces different levels of scrapping which is of prime interest in the model developed.

The stock of cars in each of the years 1994 to 1997 for the different age groups are illustrated in *Figure 4*. The peek of cars in the age 8 to 12 years illustrate the fiscal policies implemented in 1986-1987. Again it can be seen that there has been a huge increase in the acquisitions since 1994, leading to the overall general increase in the size of the car fleet.

The pattern in the figure is what should be expected (apart from the mentioned policy impacts in the 80's); the number of cars in each age group is decreasing as the cars get older.

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[3] The reason for this stratification on weight groups is that this is the closest decomposition of the cars relative to engine size. Engine sizes are the important variable when looking at energy consumption. This information, however is not contained in the basic dataset, and the work needed to make the information available is too much for this project. Work is currently undertaken at the Transport directorate in Denmark to make information on engine size available in the general database.

The pattern change slightly when we look at the different weight groups[4]. The cars in the weight category between 800 and 1000 kilos show a pattern similar to the general pattern, but the group of small, and the group of large cars differ from this general pattern. The small cars had been facing general increase in numbers until the mid 80's, after which the acquisition have been low. The number of large cars on the other hand seem to have been stable until about the same time, after which a heavy increase in acquisitions have occured. These effects can be seen in *Figure 5*.

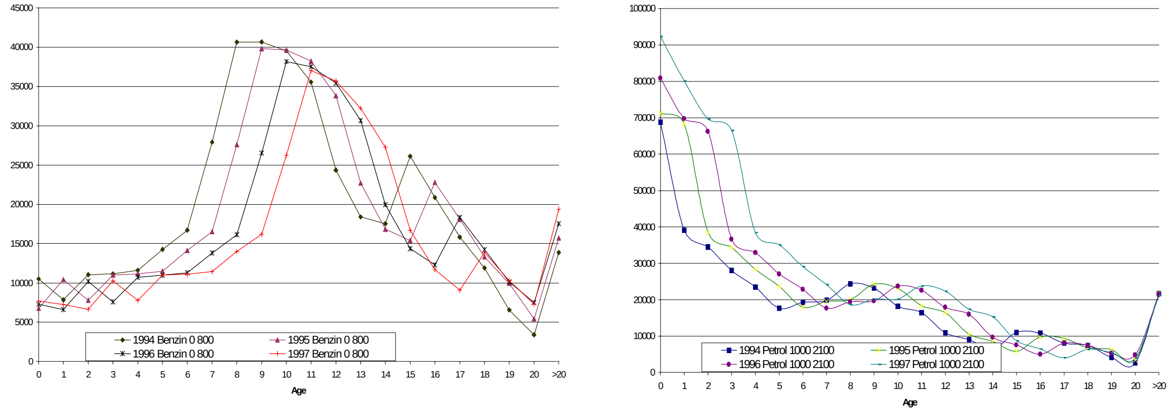


Figure 5 **Number of cars in the weight categories: less than 800 kilos and above 1000 kilos on different ages.**

The result of these effects can be seen in figure 6, where the stock of cars is decomposed into four weight groups (the group of large cars have been divided into two separate groups) for the years 1991 to 1997. It is very clear from the figure, that the rise in the overall car fleet is a result of an increasing number of large cars outweighing the decline in the number of small

a) b)

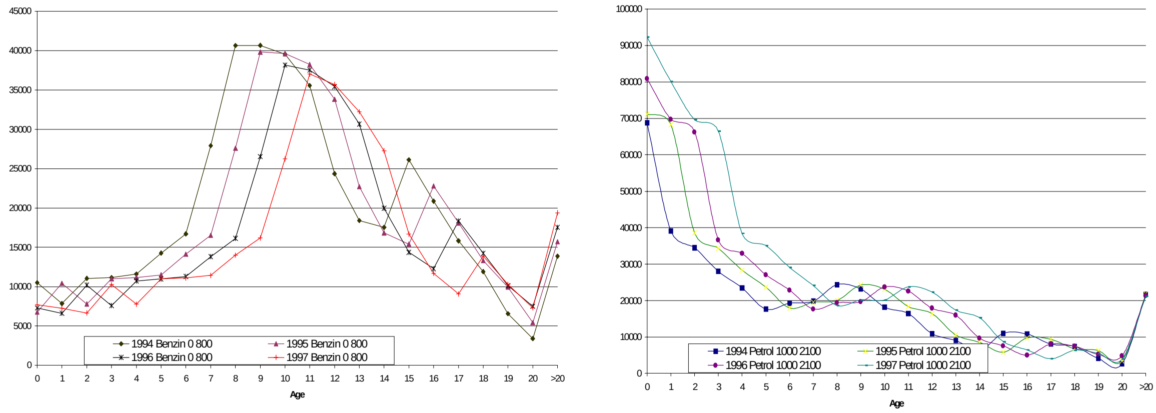


Figure 6 **The development in the number of cars in different weight categories for a) the entire fleet, and b) the acquisitions of new cars.**

[4] As the number of diesel cars is relatively low - below 5% - the distinction between petrol and diesel cars is ignored in this analysis.

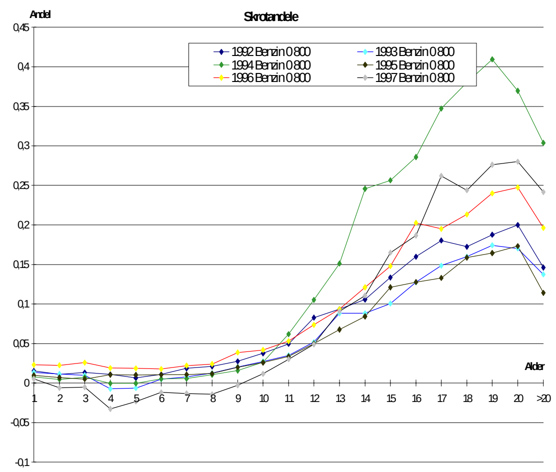
cars. The number of medium sized vehicles remain relatively stable during this period. Stressing this point further the development in acquisitions of new cars is shown in figure 6b where the general trend is that most new cars are from the group of large cars.

One immediate consequence of this is that the energy consumption and the emissions will increase, and will be comparatively larger than the emissions if every individual change of car remained in the same weight category.

Another equally important factor in determining the consequence of the change in the composition of the car fleet, is to look at which cars leave the fleet: the scrapping. Scrapping is here defined as the point where a car leaves the fleet.

The scrapping can be measured in both absolute measures and in the share of a given stock that leaves the fleet in a given period of time (here a year). Using shares the scrapping becomes independent of the actual level in number of cars in a given category, which is very

a)



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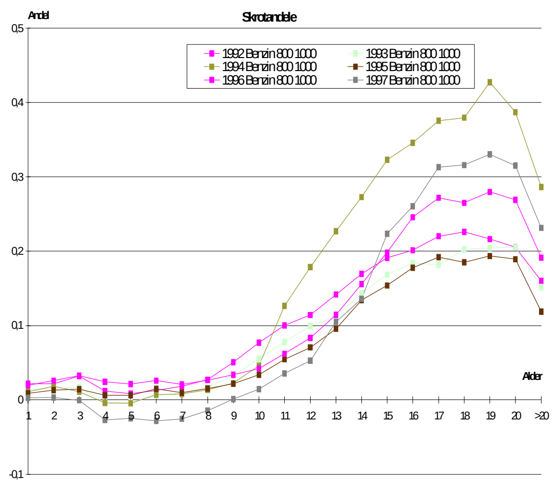

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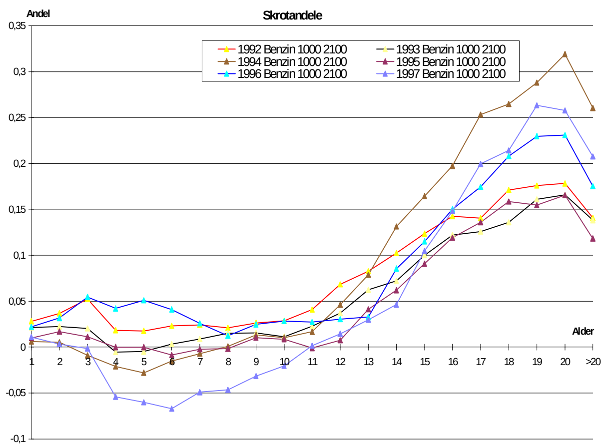

Figure 7 **The scrapping of the cars in weight groups a) 0 to 800 kg. b) 800 to 1000 kg. and c) above 1000 kg. measured as shares of the existing number of cars of the give age.**

desirable for comparing different years and for estimation purposes. Unfortunately, as will be shown later, this limits the amount of information that can be used in the estimation of the model. For the analysis in this section it is easier to look at figures using shares.

The shares of cars scrapped in 1992 to 1997 are shown in figure 7 for the different weight groups depending on the age of the cars.

The patterns in these figures are the same independent of weight. For the newest cars there are a negative scrapping indicating that used cars are imported, have been temporarily off the market, or that a given model is still sold even though it is no longer the newest variant. In the database these types of acquisitions are reported as belonging to the age group they would have been in, had they been sold as totally new cars.

As the cars grow older the share rises. One important conclusion from figure 7 is that the share of small cars scrapped are larger than the medium and large cars. This means that large cars last longer than small cars, and more importantly that the rate of change from smaller cars towards large cars will happen even faster, as the change is twofold; both through the acquisitions and through the scrapping.

It is these tendencies that should be reflected in the estimated model. At this point, interest has only been given to the problem of finding or estimating a model that gives an adequately fit to the data. As will be shown in the following, this has been quite successful.

4. The car park model

The stock of cars is made of three parts: The existing stock (T_{t-1}), the acquisitions (N_t), and the scrapping (S_t) in the following way:

$$T_t = T_{t-1} + N_t - S_t \tag{1}$$

When the stock is known, the development is hence decided from the developments in the acquisitions and scrappage. These two elements are estimated by the model developed here. It is actually estimated by two separate models: a model of acquisitions developed by Cowi consult (see Cowi, 1998), and a model of scrappage developed here. Even though there in reality are close connections between scrapping and acquisition.

From (1) it is evident that either the number of acquisitions or the number of scrapped cars become a residual if both the existing stock (T_{t-1}) and the new stock (T_t) are known. This is actually the case in the ALTRANS model complex, where the stock of cars in every period is calculated by the behavioural model. In the present model, the acquisitions therefore are the residuals.

To calculate the emissions resulting from the traffic performance it is very important to separate traffic performance on different categories of cars. In connection with the decision on which part of the development that should be calculated as a residual, it is the possibility of using cars of different ages, that is important. We will return to the problem of combining the total traffic performance with traffic performance on each category of cars later on. Cars from different years have different technologies, and hence have different energy consump-

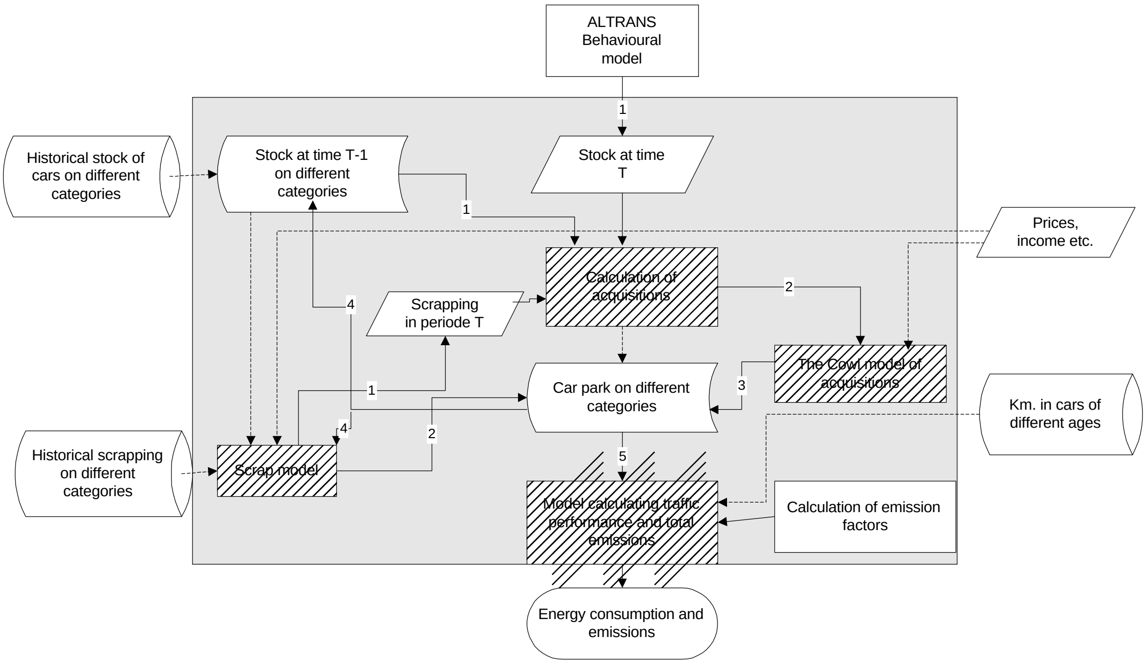

Figure 8 **The car park model.**

The tubes in the left hand side indicate data used for estimation. The shaded boxes indicate the model parts of primary interest here. Dotted lines indicate indirect input or connection.

tion; this will also be the case in the future. In fact the EU have set up a set of maximum levels of emission factors, that future cars have follow. This is why it is important to distinguish between the different age categories in the model. It is also the reason it is decided to let the acquisitions be the residual in the model. This decision is reinforced as the acquisition model developed by Cowi only distribute the number of acquisitions on approximately 1000 different makes of cars. An aggregation of this is used in the present model. The problem now is to construct a model that calculates the number of scrappings.

The development in the stock of cars in this model can be illustrated as in *Figure 8*. Following the arrows in the figure it can be seen that the model calculates the stock successively from the base year to the final forecast year.

The scrappage model

The model is estimated using the time series data described in section 3. One way of formulating the model is using ordinary least squares (or generalised least squares when the model is formulated as a general linear model[5]). This way of formulating the model have

[5] When using linear regression a large amount of different model formulations and estimation procedures are possible. A number of these have been investigated, they will however not be discussed here.

several difficulties. One obstacle is that the time series has a very short time span (only 6 years); the information from the years 1977 to 1991 is only available on aggregated levels, whereas the data from 1991 to 1992 is very detailed as described in section 3. A procedure for augmenting the data from 1977 to 1991 to the same level of detail have been investigated. The procedure gave relatively good results, but the main problem in using generated data, however good the generation method is, is that there is a big risk of systematic biases in the estimation results. The formulation that is implemented in the scrap model is the more general method of maximum likelihood estimation. With this method it is possible to use all available information without deciding prior to the estimation what systematic relations there are in the data.

The model that is to be estimated can be described in the following equations

$$S_{ijk,t} = \boldsymbol{\beta}_{ijk} X_t + \varepsilon_{ijk,t} \quad e_{ijk,t} \sim N(0,\sigma^2), \quad t = 1992,\ldots,1997 \qquad (2)$$

$$S_{..k,t} = \boldsymbol{\beta}_{..k} X_t + \varepsilon_{..k,t} \qquad (3)$$

$$S_{.j.,t} = \boldsymbol{\beta}_{.j.} X_t + \varepsilon_{.j.,t} \qquad t = 1978,\ldots,1991 \qquad (4)$$

$$S_{i..,t} = \boldsymbol{\beta}_{i..} X_t + \varepsilon_{i..,t} \qquad (5)$$

Where $S_{ijk,t}$ is the scrapping of car in category i (fuel type), j (weight) and k (age) at time t. In the same way, $S_{..k,t}$ for example, is the scrapping in category k, where the link to the disaggregated scrapping is found by summing over the relevant categories. The problem in the years 1977 to 1991 is that the disaggregated figures are unknown. $\boldsymbol{X}_t$ is a vector of exogenous variables (price indices for fuel, acquisitions, repairing costs, income etc.), $\boldsymbol{\beta}_{ijk}$ is a vector of parameters to be estimated, this vector is unique for category ijk, and finally $\varepsilon_{ijk,t}$ is a random variable, assumed independent normally distributed with mean 0 and variance to be estimated along with the other model parameters[6].

The log-likelihood function for this problem is:

$$\begin{aligned}
\ell &= \sum_{ijk}^{IJK} \sum_{t=1992}^{1997} \log\left(f_{S_{ijk}}(S_{ijk}) \right) \\
&+ \sum_{t=1978}^{1991} \left(\sum_{i}^{I} \log\left(f_{Si..}(S_{i..,t}) \right) + \sum_{j}^{J} \log\left(f_{S_{ji..}}(S_{.j.,t}) \right) + \sum_{t}^{K(t)} \log\left(f_{S_{..k}}(S_{..k,t}) \right) \right) \\
&= \ell_1 + \ell_2 + \ell_3 + \ell_4
\end{aligned} \qquad (6)$$

where $f(\cdot)$ is the density function for a given category. The function for the first part of the joint likelihood function l_1 is the usual one when using the normal distribution. The special element in the likelihood function comes from the latter three parts.

The resulting joint likelihood function is shown in an annex – the derivation of this can be found in Kveiborg (1999).

[6] Other distributions could be stipulated, but in the present model the normal distribution have been used to retain simplicity. This is actually the main objection to the maximum likelihood method, that a distribution have to be assumed prior to the estimation.

In the maximum likelihood formulation all parameters are estimated jointly. This result in the number of parameters to be equal to 120 times the number of exogenous variable in the model (because of the large number of individual equations – one for each category *ijk* – it has been decided to use the same set of explanatory variables in all 120 categories, even though a better model fit could be achieved by formulating individual equations for each category).

Actually the variance term σ^2 should also be estimated jointly with the other parameters. The way this is achieved is by finding the optimal value of the likelihood function given the parameters $\boldsymbol{\beta}$ (by simple first order differentiation the likelihood function with regard to σ and setting the result equal to zero). This expression is substituted for σ in the likelihood function.

Because of the limited number of observations it is necessary to keep the number of explanatory variables low as well. The optimal explanatory variables to have used would have been separate indices of prices etc. for the different categories. It is not possible to get information at this level of detail, instead general indices have been used. To evaluate the ability of the different model formulations, both the maximum likelihood values, likelihood ratio statistic and the R^2 statistic are used. In principle all possible formulations of the model should be estimated and the likelihood ratio statistic of the comparison of all combinations of the formulations should be evaluated. Instead a normal stepwise selection method have been implemented, backwards selecting insignificant variables to be eliminated from the model one by one until no more insignificant variables can be found. The problem in using this method is that the elimination at each step determines the selection path. It is therefore possible that a model not in the chosen path, have a better explanatory power. It is especially possible in this model as the explanatory variables (se table 1 below) tend to be highly correlated so that the use of one subset of explanatory variables could exhibit the same explanatory power as another subset. No measures have been taken to control for this correlation effect.

In table 1 the chosen selection sequence is shown, where the last three rows indicate the models evaluated in the last step of the selection[7]. This illustrate, that all of the remaining variables are significant and therefore cannot be removed without losing much of the explanation of the model. The ML values are evaluated using the χ^2 statistics, which is approximately 140 for these models. Therefore the changes in ML values in the first three steps are insignificant.

The R^2-values for all the models are quite high, and the model chosen on the basis of the likelihood ration test is actually also the model giving the highest corrected R^2 value[8]

Table 1 An illustration of the stepwise (backwards) selection procedure applied in the model estimation.

CPI: Consumer price index, LR: likelihood ratio compared with previous step, R^2-corr: R^2 value corrected for number of variables and number of parameters.

Step	Stock	Income	CPI	Acq. costs	Fuel costs	Repair costs	Dummy 94	ML-value	LR	R^2-corr

[7] It has not been possible to find a smaller model having the same explanatory power.

[8] It is actually not the model giving the highest R^2value, but the models with higher values also have a higher number of parameters to be estimated.

1	X	X	X	X	X	X	X	-13492,7		-0,55
2	X	X	X	X	X	X		-13501,3	19,2	0,24
3	X	X		X	X	X		-13547,3	92,0	0,49
4	X	X			X	X		-13587,8	91,0	0,56
5	X				X	X		-13761,1	346,6	0,55
5	X	X				X		-13688,3	201,0	0,53
5	X	X			X			-13783,9	392,2	0,54

The stock of cars in each category have been includes to fix the level scrapping. It has been tried to estimate the models without this variable, but this resulted in a large change in both ML values and R^2-values in all situations. Had the model been estimated on rates of scrapping instead of the actual levels, inclusion of this variable would not have been necessary. Unfortunately it is not possible to do this with the limited data available. This is due to the fact that the rate of scrapping in the aggregate *i,j* or *k* groups is different from the sum of the individual rates of scrapping in the *ijk*-groups: $s_{..k,t} = \sum_{ij} s_{ijk,t} = \sum_{ij} \frac{S_{ijk,t}}{T_{ijk,t-1}} \neq \frac{\sum_{ij} S_{ijk,t}}{\sum_{ij} T_{ijk,t-1}} = \tilde{s}_{..k,t}$

S indicate actual levels, and *s* indicate rates.

5. Calculation of emissions

The final part of the calculations are to determine the emissions and the energy consumption from the transport estimated in the behavioural model and the estimated car park from the car park model. Again the behavioural model only determines the absolute level of driven kilometers. To be able to link this to the individual cars, information from the Danish Road Directorate (Ekman and Kristensen, 1998) on the average distance a car of age *k* drives in one year. This figure is multiplied by the number of cars in the given age category and then corrected using the ALTRANS estimate. This is a very mechanical way of doing this calculation, unfortunately it is at this point not possible to make a more specific relation between the individual behaviour and the overall car park. This is a thing that will be looked upon in the future.

At this point more effort has been put into determining the emission factors for the different categories of cars. This work is more elaborately described in Winther (1999) and Kveiborg (1999).

When cars are getting older their emissions rise. This is most important for the cars with a catalyst exhaust purifier, which become worn out the more the car is used. A degradation factor for this is known from the Copert database (Ahlvik et al. 1997). This factor is dependant on the km. driven. A general formula for the relation between age (Y) and km. (X) is stipulated from information given from the Road Directorate:

$$X = \begin{cases} -405.033 * Y^2 + 25.800 * Y + 4194, & Y < 18 \\ -405.033 * 18^2 + 25.800 * 18 + 12.525 * (p - Y - 18), & Y > 17 \end{cases}$$

The emission factor for emission component *i* is described by the following relation:

$$e_{i,X} = \frac{e_{i,50}}{(f_{i,80} - 1) \cdot \frac{50.000}{80.000} + 1} \cdot (\frac{f_{i,80} - 1}{80.000} \cdot X + 1)$$

where $f_{i,80}$ expresses how much the emissions rise when a car has driven 80.000 km. This factor changes depending on the emission factor. The emission factor $e_{i,50}$ is known from the Copert database. It is also this factor that the EU has set a standard for, both regarding vehicles made today, but also for future vehicles. In this way a kind of technological progress is included in the model. It is now evident why it is necessary to distinguish between the different age categories for the cars, - it is actually the year of first registration that is important here, if both the degradation effect and the technological progress are to be included in the model.

Combining the two expressions give the emission factor to be used in the model. This factor is however only for the so-called hot emissions, where the car engine has been running for some time, and the vehicle has driven at least 4 km. When the car has a cold engine, the emissions are higher. Therefore a cold engine component is added for each trip in the car. This is however only the case for catalyst cars, and not for conventional cars.

The problem in relation to this is that the actual number of trips in each car is not known. From the behavioural model only the total number of trips is known. Another important problem in relation to this is that the knowledge to combine the information on the length of each trip with which car has been used on this particular trip is not available. A procedure using weighted averages of the different types of trips from the behavioural model have been used given one average cold emission factor for each of the 120 car type categories. The total emissions are the sum of the hot and the cold emissions.

6. Concluding remarks

In this paper a model that is a small part of the ALTRANS has been presented. The general procedure in the model is quite mechanical. However the individual elements in this mechanical model are not so simple. The three main components are a model for acquisitions, a model for scrapping and finally a model calculating emissions from the stipulated car park and traffic performance.

The model described in the paper is currently being implemented in the overall model complex ALTRANS. The model has not yet been used for calculations of the kind described in the paper. In the near future, the model will be used intensely for a line of different scenarios.

It is evident that a lot of different improvements could be made to the model. Some of these will be made during the use of the model in the future.

One of the improvements that could most certainly improve the model is a closer connection between the behavioural elements in choice of car described in the behavioural model, and the model calculating the future composition of the car park, this is the model described here. Another equally interesting improvement is to combine the detailed knowledge of the different trips contained in the national travel survey and used in the calculations in the behavioural model, with the calculation of cold emissions in the car park/emission model. These improvements are not an easy task however, which is also the reason this has not been implemented yet.

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Annex

Final joint log-likelihood function.

$$\ell = \ell_1 + \ell_2 + \ell_3 + \ell_4 \tag{7}$$

$$\ell_1 = A_1 - \frac{1}{2\sigma^2}\sum_{ijk}(\mathbf{T}_{ijk} - \mathbf{X}\boldsymbol{\beta}_{ijk})^T(\mathbf{T}_{ijk} - \mathbf{X}\boldsymbol{\beta}_{ijk}) \tag{8}$$

$$\ell_2 = A_2 - \frac{1}{2J\sigma^2}\sum_{i}\left(\frac{1}{\mathbf{k}}(\mathbf{T}_{i..} - \mathbf{X}\boldsymbol{\beta}_{i..})\right)^T(\mathbf{T}_{i..} - \mathbf{X}\boldsymbol{\beta}_{i..}) \tag{9}$$

$$\ell_3 = A_3 - \frac{1}{2I\sigma^2}\sum_{j}\left(\frac{1}{\mathbf{k}}(\mathbf{T}_{.j.} - \mathbf{X}\boldsymbol{\beta}_{.j.})\right)^T(\mathbf{T}_{.j.} - \mathbf{X}\boldsymbol{\beta}_{.j.}) \tag{10}$$

$$\ell_4 = A_4 - \frac{1}{2IJ\sigma^2}\sum_{k}^{k(t)}(\mathbf{T}_{..k} - \mathbf{X}\boldsymbol{\beta}_{..k})^T(\mathbf{T}_{..k} - \mathbf{X}\boldsymbol{\beta}_{..k}) \tag{11}$$

$$A_1 = -N_1 IJK\log(\sqrt{2\pi\sigma^2}),\ A_2 = -N_2 I\log(\sqrt{2\pi J\sigma^2}) - I\sum_{t=1978}^{1991}\log\sqrt{k(t)}$$

$$A_3 = -N_2 J\log(\sqrt{2\pi I\sigma^2}) - J\sum_{t=1978}^{1991}\log\sqrt{k(t)},\ A_4 = -K(t)\log(\sqrt{2\pi IJ\sigma^2})$$

The ***k***-vector is a vector of the number of observations of the cars in age group *k*. In the same way *k(t)* and *K(t)* are the *t*'th element of the vector, and the sum of the individual elements. Had the time series for the age category been of full length, the number of *k*-categories would have entered the formulation in the same way as the *j* and *i* categories.